**RFT 9.0: The Adaptive Scalaron – Unifying Quantum Waves, Dark Matter, and Gravity**

**Introduction**

Astrophysical and cosmological observations demand a form of “dark” gravitating matter, yet decades of searches have not revealed a conventional particle. At the same time, modifications to gravity on galactic scales (e.g. MOND-like phenomenology) hint that our understanding of gravity itself might be incomplete. A promising theoretical approach is that **dark matter and modified gravity phenomena could be two manifestations of a single underlying field**, whose behavior *adapts* to its environment​[scipost.org](https://scipost.org/submissions/2109.10928v1/#:~:text=suitable%20choice%20of%20the%20superfluid,studied%20physics%20of%20superfluidity). In this framework – developed in the Relativistic Field Theory (RFT) cosmology program – a **scalar field** dubbed the **“adaptive scalaron”** serves as this unifying agent. This scalaron field can manifest as a wave-like quantum condensate on cosmic scales, mimic cold dark matter in galaxies, and modify geometric gravity in the strong-field regime, all depending on environmental conditions.

Recent advances in scalar field cosmology support this vision. **Ultralight bosonic fields** (with masses $m \sim 10^{-22}$–$10^{-20}$ eV) have emerged as viable dark matter candidates that form macroscopic quantum states on kiloparsec scales​[nature.com](https://www.nature.com/articles/nphys2996#:~:text=galaxies%2C%20with%20only%20one%20free,%E2%89%B2%2013%20in%20our%20simulations). Such fields can exhibit wave interference and solitonic cores inside galaxies​[nature.com](https://www.nature.com/articles/nphys2996#:~:text=galaxies%2C%20with%20only%20one%20free,%E2%89%B2%2013%20in%20our%20simulations), addressing small-scale structure puzzles (like the cusp–core problem) that plague standard cold dark matter​[nature.com](https://www.nature.com/articles/nphys2996#:~:text=The%20conventional%20cold,achieve%20cosmological%20simulations%20of%20this). In parallel, scalar-tensor modifications of gravity (e.g. $f(R)$ gravity and chameleon fields) show that a scalar field coupled to curvature can reproduce galaxy dynamics without invoking heavy halos​[aanda.org](https://www.aanda.org/articles/aa/pdf/2014/01/aa21061-13.pdf#:~:text=Aims,its%20context%20and%20applied%20it)​[aanda.org](https://www.aanda.org/articles/aa/pdf/2014/01/aa21061-13.pdf#:~:text=Results,of%20galaxies%2C%20show%20evident%20correlations). The adaptive scalaron hypothesis merges these lines of thought: **a single scalar field whose effective mass and dynamics change with local density**, yielding quantum wave behavior in low-density voids and behaving like a classical gravitating mass in high-density regions. **Dark matter and modified gravity become simply different phases of one field**​[scipost.org](https://scipost.org/submissions/2109.10928v1/#:~:text=suitable%20choice%20of%20the%20superfluid,studied%20physics%20of%20superfluidity).

This document, designated *RFT 9.0: The Adaptive Scalaron*, consolidates theoretical developments and recent findings on this unifying framework. We present a formal theoretical model of the adaptive scalaron field and explore how it interpolates between quantum, classical, and geometric (curvature-coupled) regimes of gravity. We define the scalaron and its role in RFT cosmology (§2); examine its behavior in three key gravitational regimes – **fuzzy quantum wave**, **decoherent classical**, and **gravitational collapse** (§3); analyze its connection to entropy production and the cosmological arrow of time (§4); and discuss how it bridges multiple domains of theory, from dark matter phenomenology to modified gravity and twistor-geometric formulations (§5). In §6 we propose observational signatures that could test this model, including solitonic core oscillations, tidal disruption effects, collapse-generated wave signals, and black hole “hair.” Finally, in §7 we outline open theoretical questions and future directions (e.g. scalaron–curvature couplings, torsion models, collapse dynamics, and phase transitions). Throughout, we adopt an academic tone and unified notation, treating the scalaron as a field $\phi$ with a potential $V(\phi)$ and coupling functions chosen to realize the adaptive behavior. All equations are given in units where $c=\hbar=1$ (unless otherwise noted). Our goal is to provide a self-contained theoretical reference for this adaptive scalaron paradigm, which aspires to **unify dark matter and modified gravity within a single relativistic field theory**.

**The Adaptive Scalaron in RFT Cosmology**

**Definition and Origin:** *Adaptive scalaron* refers to a cosmological scalar field $\phi(x)$ whose properties (effective mass, coupling to matter/curvature, quantum coherence length) depend on the local gravitational environment. In low-density cosmic voids, $\phi$ remains light and coherent, while in high-density regions (galactic cores, stellar systems) it becomes massive or couples strongly to gravity, thus “hiding” its wave nature. The term “scalaron” is borrowed from $f(R)$ gravity, where a new scalar degree of freedom emerges from extending the Ricci scalar $R$ in the action​[inspirehep.net](https://inspirehep.net/files/2e446914ecf6a036ff82d5de6c35a1dd#:~:text=that%20power,%282015%29%20used). In Starobinsky’s $R^2$ inflation, for example, the **scalaron** is the scalar field equivalent of the $R^2$ term driving inflation​[inspirehep.net](https://inspirehep.net/files/2e446914ecf6a036ff82d5de6c35a1dd#:~:text=that%20power,%282015%29%20used). Here we generalize the notion: the adaptive scalaron is a single field postulated to pervade the cosmos, playing multiple roles – as dark matter in some contexts and as a modifier of gravity in others.

**RFT Cosmology Framework:** This idea is developed in the Relativistic Field Theory (RFT) cosmology framework​file-pbs5tcrmsvz7ndprsed51h. RFT cosmology posits that what we call “dark matter” and even “dark energy” may arise from one (or a few) fundamental fields within an extended gravitational action. In earlier RFT studies, an ultralight scalar was identified as a compelling dark matter candidate​file-c422leaz9pf3pftzffrfgv. In particular, a **“fuzzy” scalaron identified with an ultralight axion-like particle** was hypothesized​file-c422leaz9pf3pftzffrfgv. Such a field could naturally arise from high-energy theory: for instance, as a **pseudo-Nambu–Goldstone boson (axion)** from a broken global symmetry in the early universe​file-c422leaz9pf3pftzffrfgv​file-c422leaz9pf3pftzffrfgv. These axion-like scalarons have extremely small masses ($m \sim 10^{-22}$ eV) generated by non-perturbative effects, and are stable due to an approximate shift symmetry​file-c422leaz9pf3pftzffrfgv. They carry a **calculable relic abundance** set by the misalignment mechanism in the early universe​file-c422leaz9pf3pftzffrfgv, making them excellent dark matter candidates. In fact, string theory predicts a whole “axiverse” of ultralight axions; it is plausible one of the lightest has $m \sim 10^{-22}$ eV, right in the range for fuzzy dark matter​file-c422leaz9pf3pftzffrfgv. RFT cosmology 8.x studies pinpointed this **ultralight axion scalaron** as a unification candidate, with a Lagrangian of the form:

S=∫d4x−g[116πGR+12(∂μϕ)2−V(ϕ)]+Sint[ϕ,gμν,Ψi],S = \int d^4x \sqrt{-g} \left[ \frac{1}{16\pi G}R + \frac{1}{2}(\partial\_\mu \phi)^2 - V(\phi) \right] + S\_\text{int}[ \phi, g\_{\mu\nu}, \Psi\_i ],S=∫d4x−g​[16πG1​R+21​(∂μ​ϕ)2−V(ϕ)]+Sint​[ϕ,gμν​,Ψi​],

where $V(\phi)$ is a shallow potential (e.g. $m^2 \phi^2/2$ or a periodic axion potential), and $S\_\text{int}$ encodes possible non-minimal interactions of $\phi$ with curvature $R$ or matter fields $\Psi\_i$ to enable environment-dependent effects. The hallmark of the scalaron is that its **effective mass $m\_\text{eff}(\phi,\rho)$ and coupling strength vary with ambient mass density $\rho$ or curvature**. In a low-density region, $m\_\text{eff}$ is tiny, so $\phi$ coherently oscillates over large scales and mediates long-range forces; in a high-density region, $m\_\text{eff}$ grows (a “chameleon” mechanism​[aanda.org](https://www.aanda.org/articles/aa/pdf/2014/01/aa21061-13.pdf#:~:text=Aims,its%20context%20and%20applied%20it)), so $\phi$ is short-range and effectively pressureless.

**Unifying Dark Matter and Modified Gravity:** In this single-field picture, the scalaron underlies both the dark matter phenomena in cosmic structure and any apparent modifications to gravity in regimes where dark matter is scarce. This is conceptually illustrated by analogies to *phase transitions*: **the scalaron has multiple phases**. In one phase (low-density), it behaves like a quantum condensate with wave-like (or superfluid) properties; in another phase (intermediate-density), it behaves like a classical collisionless dust; in yet another (extreme density/curvature), it becomes strongly coupled to geometry, altering the effective gravitational law. Notably, **Khoury’s theory of superfluid dark matter** embodies a similar paradigm: galactic halos of an axion-like particle condense into a superfluid core whose phonon excitations give rise to MOND-like modified gravity, while outside the core the particles are normal cold dark matter​[scipost.org](https://scipost.org/submissions/2109.10928v1/#:~:text=In%20these%20lectures%20I%20describe,studied%20physics%20of%20superfluidity). *“Thus the dark matter and modified gravity phenomena represent different phases of a single underlying substance, unified through the rich physics of superfluidity.”*​[scipost.org](https://scipost.org/submissions/2109.10928v1/#:~:text=suitable%20choice%20of%20the%20superfluid,studied%20physics%20of%20superfluidity) Although the scalaron in RFT 9.0 is an ultralight ($\sim10^{-22}$ eV) field (distinct from the $m\sim 1$ eV scale of superfluid DM models​[scipost.org](https://scipost.org/submissions/2109.10928v1/#:~:text=In%20these%20lectures%20I%20describe,different%20phases%20of%20a%20single)), the principle is analogous: **one field, multiple behaviors**. The scalaron can form condensate cores that produce effective long-range forces on baryons (mimicking modified gravity), while most of its mass still behaves as diffuse dark matter on larger scales. Environmental dependence is key: the field’s behavior is *adaptive* – a dense concentration of baryonic or dark mass can trigger the scalaron to transition from one regime to another (for example, suppressing its gradient pressure and acting like a classic gravitational potential). In modified gravity terms, the scalaron acts as an extra degree of freedom that is active on galactic scales but screened in the solar-neighborhood, much like chameleon fields that **vary with local matter density**​[aanda.org](https://www.aanda.org/articles/aa/pdf/2014/01/aa21061-13.pdf#:~:text=Aims,its%20context%20and%20applied%20it).

To summarize, the adaptive scalaron is a cosmological scalar field motivated by high-energy theory (e.g. an axion from the string axiverse)​file-c422leaz9pf3pftzffrfgv that **constitutes the dark matter** while simultaneously providing a mechanism for **environment-dependent modifications of gravity**. In the next sections, we detail how this single scalar field behaves across different gravitational regimes and how it provides a continuous interpolation between quantum, classical, and relativistic-gravity domains.

**Regimes of Scalaron Dynamics Across Scales**

A central thesis of the adaptive scalaron framework is that a single field $\phi$ can reproduce the appropriate behavior in *three distinct gravitational regimes*: (i) the **fuzzy quantum wave regime** prevalent in low-density environments (or early linear universe), (ii) the **decoherent classical regime** in virialized galactic and cluster halos, and (iii) the **collapse/black-hole regime** in extreme overdensities. We now characterize the scalaron’s behavior in each regime, highlighting how the field’s equations of motion reduce to the expected physics.

**Fuzzy Wave Regime (Quantum Coherent Phase in Voids)**

In the most diffuse environments – cosmic voids or the outskirts of halos where the ambient density of matter is extremely low – the scalaron remains in a **quantum wave-dominated regime**. Here $\phi$ can be thought of as a *giant coherent wavefunction* oscillating in its potential. The de Broglie wavelength $\lambda\_{\rm dB}$ of the field (inversely proportional to momentum) is huge, often kiloparsecs, exceeding the size of small dark matter structures. As a result, the scalaron does not fragment into particles but forms a **Bose–Einstein condensate (BEC) on cosmological scales**​[nature.com](https://www.nature.com/articles/nphys2996#:~:text=The%20conventional%20cold,achieve%20cosmological%20simulations%20of%20this). The uncertainty principle (quantum pressure) counters gravity below a Jeans scale, preventing arbitrarily small clumps​[nature.com](https://www.nature.com/articles/nphys2996#:~:text=The%20conventional%20cold,achieve%20cosmological%20simulations%20of%20this). This regime corresponds to the idea of **“fuzzy dark matter”** or **wave dark matter (ψDM)** in the literature​[nature.com](https://www.nature.com/articles/nphys2996#:~:text=The%20conventional%20cold,achieve%20cosmological%20simulations%20of%20this).

**Wave Mechanics:** In this phase, the dynamics of $\phi$ are well-described by the coupled Schrödinger–Poisson (SP) or Gross-Pitaevskii equations. The scalaron’s gravitational potential $\Phi$ and wavefunction $\psi \sim e^{-i m t}\phi$ obey:

iℏ∂ψ∂t=−ℏ22m∇2ψ+mΦψ,∇2Φ=4πG(ρb+m∣ψ∣2),i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + m \Phi \psi, \qquad \nabla^2 \Phi = 4\pi G (\rho\_b + m|\psi|^2),iℏ∂t∂ψ​=−2mℏ2​∇2ψ+mΦψ,∇2Φ=4πG(ρb​+m∣ψ∣2),

where $|\psi|^2$ gives the scalaron density and $\rho\_b$ is any ordinary (baryonic) density present. In voids, $\rho\_b \approx 0$. The key feature is that $\psi$ remains *single-valued with a well-defined global phase*, i.e. a coherent state. Large-scale structure formation proceeds similarly to cold dark matter on large scales, but on small scales the wave nature becomes evident. Indeed, cosmological simulations of wave dark matter show that on **large scales the structure is indistinguishable from CDM**, but **inside halos the behavior differs markedly**​[nature.com](https://www.nature.com/articles/nphys2996#:~:text=galaxies%2C%20with%20only%20one%20free,%E2%89%B2%2013%20in%20our%20simulations). Quantum interference effects dominate the internal structure of halos in this regime.

**Solitonic Cores:** A hallmark of this fuzzy regime is the formation of **solitonic cores** – self-supported, stationary wave solutions that sit in the centers of gravitational potential wells. As a halo grows by accretion or mergers, the central region of the scalaron field can relax into the lowest-energy eigenstate (a soliton). **Simulations demonstrate that quantum interference of the coherent field naturally produces a solitonic core surrounded by a halo of wave interference “granules.”** In a representative result by Schive *et al.*, *“quantum interference forms solitonic cores surrounded by extended halos of fluctuating density granules.”*​[nature.com](https://www.nature.com/articles/nphys2996#:~:text=galaxies%2C%20with%20only%20one%20free,%E2%89%B2%2013%20in%20our%20simulations) The soliton is a Bose–Einstein condensate core where quantum pressure (from the $\nabla^2\psi$ term) balances gravity. Its density profile is well-fit by $\rho\_{\rm soliton}(r) \propto [1 + 0.091 (r/r\_c)^2]^{-8}$, with a core radius $r\_c$ scaling inversely with the core mass and the particle mass. Surrounding the soliton, the remainder of the halo consists of a chaotic interference pattern of excited states – these manifest as time-dependent, spatially granular density fluctuations often described as a “quantum fog.” **The entire halo can thus be viewed as a standing wave (core) plus a superposition of traveling waves (envelope).** This picture is vividly supported by simulations: for instance, a fuzzy dark matter halo shows a dense solitonic center and an envelope akin to an NFW profile but with continual small-scale fluctuations​[osti.gov](https://www.osti.gov/biblio/1851580#:~:text=We%20report%20that%20a%20fuzzy,As%20the).

**Examples:** In a dwarf galaxy-size halo (virial mass $\sim10^9 M\_\odot$), an ultralight scalaron ($m \sim 10^{-22}$ eV) would form a core of radius a few hundred parsecs with a density $\sim10^2$–$10^3 M\_\odot/\text{pc}^3$, much shallower than a CDM cusp, addressing the core–cusp problem​[nature.com](https://www.nature.com/articles/nphys2996#:~:text=The%20conventional%20cold,achieve%20cosmological%20simulations%20of%20this). In cosmic voids, where large-scale potential wells are absent, the scalaron field may remain almost uniform, with only gentle acoustic oscillations. One can imagine a void as filled with a very low-amplitude, nearly homogeneous $\phi$ field oscillating at its natural frequency $m$; effectively, a patch of vacuum with a cold axion background. In such regions the field’s quantum coherence is maximal (phase correlations over Mpc scales). There could even exist **interference patterns on intergalactic scales**, e.g. filamentary standing-wave nodes, though these would be subtle. Overall, the fuzzy wave regime ensures that on the largest scales the scalaron behaves like a smooth fluid (preserving the successes of $\Lambda$CDM on CMB and galaxy clustering), while on small scales it introduces quantum pressure that prevents over-densification of dark matter and naturally forms cores​[nature.com](https://www.nature.com/articles/nphys2996#:~:text=The%20conventional%20cold,achieve%20cosmological%20simulations%20of%20this).

**Decoherent Classical Regime (Galaxies and Clusters)**

As one considers environments of higher density – for example, the interior of large dark matter halos such as the Milky Way or clusters of galaxies – the scalaron’s behavior transitions toward that of **classical, pressureless dark matter**. This is the regime of a **decohered scalaron**, where the field’s phase coherence is largely lost due to complex dynamics and interactions. Gravitational collapse and virialization excite many modes of the field, populating a quasi-thermal ensemble of wave states rather than a single coherent ground state. In plainer terms, **the scalaron field in a galaxy behaves as a collection of particles** for most purposes, even though fundamentally it is a wave.

**Core–Halo Structure:** Importantly, the transition is not total – the innermost region of halos can remain in the quantum solitonic state, but the outer regions become effectively classical. High-resolution studies show a **distinct “core–halo” bifurcation** in fuzzy dark matter halos: *“a fuzzy dark matter halo consists of a soliton core close to the center and a Navarro-Frenk-White–like density profile in the outer region.”*​[osti.gov](https://www.osti.gov/biblio/1851580#:~:text=We%20report%20that%20a%20fuzzy,As%20the) The inner soliton (ground state) is the vestige of quantum coherence, while the outer halo is comprised of excited states that have decohered into a virialized cloud. The outer density profile approximates the NFW form seen in CDM, meaning the scalaron reproduces the successful empirical profile of large halos beyond the core radius​[osti.gov](https://www.osti.gov/biblio/1851580#:~:text=We%20report%20that%20a%20fuzzy,As%20the). In effect, **outside the core the scalaron’s quantum pressure is negligible** and the field behaves as if it were pressureless cold matter. This is why large-scale and high-density structures (like clusters) are not disrupted by quantum effects – their de Broglie wavelength is tiny compared to the system, so one recovers the classical limit.

**Decoherence Mechanism:** The loss of coherence can be understood through wave interference and coupling to environment. When many waves overlap with uncorrelated phases (e.g. during hierarchical merging of subhalos, or interaction with baryonic potential variations), the relative phase information becomes scrambled. One can treat different patches of the scalaron field as having independently evolving phases once separated by nonlinear dynamics (much like how quantum phases decohere when a system becomes entangled with a chaotic environment). Quantum decoherence is effectively an entropy-increasing process – the initially “pure” state of the field (low entropy, single phase relation) becomes a “mixed” state with higher entropy (randomized phases)​[en.wikipedia.org](https://en.wikipedia.org/wiki/Arrow_of_time#:~:text=match%20at%20L436%20decoherence%20is,In%20the%20language). Practically, this means the interference fringes in the halo act like pseudoparticles. They do not maintain a stable phase relation and instead behave like a **cloud of collisionless DM pseudoparticles**.

For a given halo, one useful picture is the **eigenmode decomposition**: the halo’s scalar field can be expanded in a set of eigenfunctions of the gravitational potential. The lowest mode is the soliton; higher modes correspond to excited, higher-energy states. Due to gravitational interactions, these modes are continually exchanging energy (through interference and tidal perturbations), so any initial phase alignment dissipates. Li, Hui & Yavetz (2021) showed that the phenomena of **soliton core oscillations and random walks** can be explained by interference of the ground state with a spectrum of excited states​[osti.gov](https://www.osti.gov/biblio/1851580#:~:text=Previous%20investigations%20found%20that%20the,of%20the%20excited%20states%2C%20the). As long as significant excited state amplitude is present, the core will oscillate and move around, indicating the system is not in a single coherent eigenstate but a superposition​[osti.gov](https://www.osti.gov/biblio/1851580#:~:text=of%20numerical%20simulations%2C%20we%20show,As%20the). Over time, one expects some phase averaging; indeed, if a subhalo is stripped of its outer layers (reducing excited state amplitude), the soliton core’s oscillations diminish, approaching a more steady state​[osti.gov](https://www.osti.gov/biblio/1851580#:~:text=,deducing%20constraints%20from%20stellar%20heating). This is essentially **decoherence in action**: removing the “environment” (outer halo waves) leaves the core closer to a pure state.

**Effective Dust Behavior:** In the bulk of a galaxy or cluster halo, the scalaron can be treated as a classical **pressureless fluid** on macroscopic scales. Cosmologically, it means structure formation with scalaron dark matter will follow the same evolution as $\Lambda$CDM at late times – a key requirement for any dark matter model. The halo mass function, large-scale clustering, and dynamics of clusters (massive enough that their core radius is much smaller than their virial radius) should remain as in CDM. Observers would infer “cold dark matter” behavior in lensing and dynamics of clusters. In galaxies, the difference appears mainly in the inner kpc or so (where a core replaces a cusp). At larger radii, fits to rotation curves under scalaron dark matter are similar to CDM (aside from slightly lower concentration for low-mass halos due to the suppression of small-scale power)​[nature.com](https://www.nature.com/articles/nphys2996#:~:text=The%20conventional%20cold,achieve%20cosmological%20simulations%20of%20this). The scalaron thus passes the basic phenomenological tests: it forms structure on large scales nearly identical to CDM​[nature.com](https://www.nature.com/articles/nphys2996#:~:text=galaxies%2C%20with%20only%20one%20free,%E2%89%B2%2013%20in%20our%20simulations), and on small scales it naturally fixes some CDM problems by virtue of its quantum-derived core and the dearth of very low-mass halos (discussed further in §5.1).

It is worth noting that as the field enters this classical regime, the notion of a single field still holds – we are not switching to a different substance, just a different behavior of $\phi$. One could in principle recover the classical evolution by taking the Wigner transform of the wavefunction to define a pseudo phase-space density $f(\mathbf{x}, \mathbf{v})$. In the classical regime, $f$ approximately obeys the collisionless Boltzmann equation (or Vlasov–Poisson), just as CDM N-body particles would. Therefore, simulations of scalaron dark matter on cluster scales show **no significant deviation from CDM predictions** for large-scale structure​[nature.com](https://www.nature.com/articles/nphys2996#:~:text=galaxies%2C%20with%20only%20one%20free,%E2%89%B2%2013%20in%20our%20simulations). Only by looking at fine-grained structure (e.g. the granularity of the halo or the oscillation of the core) would one notice that the underlying dark matter is a wavy scalar field and not literal point particles.

In summary, the **decoherent regime** is where the adaptive scalaron acts as standard dark matter. It **coexists** with the quantum regime in every halo (quantum core + classical envelope) and ensures continuity with well-tested gravitational physics in systems like galaxy clusters, while still providing distinctive differences at smaller scales (galactic cores, satellites) that can test the model.

**Collapse and Black Hole-Bound Regime (Strong-Gravity Limit)**

The third regime occurs in regions of **extreme density or gravitational potential**, where even the classical dark matter behavior transitions – namely, when the scalaron forms compact objects approaching the Schwarzschild radius scale. This can happen, for instance, if a scalaron solitonic core grows beyond a certain critical mass and undergoes gravitational collapse. In the universe, such situations might arise in the centers of large galaxies or in collapsing primordial overdensities. The end result is the formation of a **black hole (BH)**, but with unique features stemming from the scalaron’s field nature. We refer to this as the **collapse/BH-bound regime**, characterized by **soliton instability, scalar radiation, and transient “hair” before settling into a black hole**.

**Soliton Instability and Collapse:** A self-gravitating boson (scalar) configuration is stable only up to a maximum mass. For a free (non-self-interacting) scalaron star (often called a boson star or axion star), there is a well-known **Kaup limit**: $M\_{\rm max} \approx 0.633, M\_\text{Pl}^2/m$ (where $M\_\text{Pl}$ is the Planck mass and $m$ the particle mass)​[link.springer.com](https://link.springer.com/article/10.1140/epjc/s10052-019-6940-z#:~:text=,interaction%20with%20potential%20%5C%28V%28%5Cphi). For $m\sim 10^{-22}$ eV, this gives an enormous $M\_{\rm max} \sim 10^{12} M\_\odot$ – interestingly, around the scale of a large galaxy’s dark matter halo. However, in practice, environmental factors and self-interactions alter this number. If the scalaron has even tiny self-interactions (as an axion would), an **“axion star”** can become unstable at much lower masses. Attractive self-interactions (like the cosine axion potential) tend to decrease the stability limit, causing collapse at lower masses via a bosenova-type instability​[link.springer.com](https://link.springer.com/article/10.1140/epjc/s10052-019-6940-z#:~:text=formation,in%20the%20case%20of%20dense). Repulsive self-interactions (e.g. a $\lambda \phi^4$ term with $\lambda>0$) increase the max mass considerably​[link.springer.com](https://link.springer.com/article/10.1140/epjc/s10052-019-6940-z#:~:text=concluded%20that%20in%20the%20free,28%5D%29.%20On%20the), potentially preventing collapse except in extremely massive configurations.

We posit that for the cosmologically relevant scalaron, the parameters are such that **halo cores in most galaxies are stable (sub-critical), but in extreme cases (perhaps at centers of massive galaxies or during mergers) a core can accumulate above the critical threshold**. When that threshold is crossed, the solitonic core can no longer support itself against gravity. What follows is a complex **gravitational collapse**. Numerical studies of axion star collapse show a two-stage process: first a mild collapse that triggers partial expulsion of mass (an “axion nova”), then – if enough mass remains – a final collapse to a BH​[link.springer.com](https://link.springer.com/article/10.1140/epjc/s10052-019-6940-z#:~:text=mass%2C%20leading%20to%20an%20instability,6). In the first stage, as the core compresses, its central density rises rapidly and the field oscillations become relativistic. The scalar field can begin radiating away kinetic energy in the form of scalar waves (analogous to how a collapsing BEC can eject particles). This **scalar radiation** carries away some of the mass and prevents an immediate total collapse. However, if the mass is well above critical, the expulsion is not enough to halt collapse. Eventually, a black hole forms at the center.

**Black Hole Formation and Transient Scalar Hair:** When the black hole forms, most of the scalaron’s mass within the event horizon simply becomes part of the black hole (increasing its mass). In classical GR, a stationary black hole cannot hold onto a free scalar field – the no-hair theorem dictates that a minimally-coupled scalar either falls in or radiates away. However, **during and shortly after the collapse, one can have “transient scalar hair.”** This refers to scalar field configurations outside the horizon that are non-zero for some time. For example, some of the scalar field that did not cross the horizon immediately might linger as a cloud or as outgoing waves. Because the system was highly coherent (a soliton) just prior to collapse, it is plausible that a portion of $\phi$ ends up in quasi-bound states around the nascent black hole (similar to quasi-normal modes). Over time, this hair will dissipate: massive scalar field modes will radiate their energy either into the black hole (settling inside the horizon) or to infinity as scalar (or gravitational) radiation. But the **decay can be slow** if the field is massive and gravitationally bound – in effect, the black hole + scalar system might form a gravitational atom (like the proposed axion clouds around spinning black holes via superradiance). In our case, the black hole is formed *from* the scalar, so initially there is a lot of scalar around. This scenario is ripe for rich dynamics: the scalar cloud can undergo oscillations, tidal disruption by the hole’s gravity, and resonant excitations.

An intriguing possibility is if the scalaron has self-interactions, the collapsing object might not directly produce a quiet black hole but instead an **explosive event**. There are analogies to condensed matter: a collapsing axion Bose star might undergo bosenova bursts observed as intense radiation (in analogy to laboratory BEC collapse due to attractive interactions). For cosmological scalaron, this could translate to a **burst of scalar radiation and high-frequency gravitational waves** at the moment of collapse.

**Gravitational Wave Emission:** The collapse of a scalaron core into a BH is a highly dynamic process, and like any asymmetric collapse, it should emit **gravitational waves (GWs)**. However, the frequency of these waves is set by the timescale of the collapse – which for such light scalars can be extremely high frequency (kHz to GHz). For instance, one analysis suggests that axion clumps can produce “fast gravitational wave bursts” with frequencies on the order of $\frac{1}{2}m/\pi$ (in physical units)​[link.aps.org](https://link.aps.org/doi/10.1103/PhysRevD.104.103009#:~:text=%28FRBs%29,more%20evidence%20for%20the%20axion). For $m=10^{-22}$ eV, this is $\sim 10^{-7}$ Hz – actually very low (months period) if taken literally. But if smaller, denser clumps ($m$ effectively larger or collapse happening on sub-second times) are considered, frequencies in kHz range could occur​[link.aps.org](https://link.aps.org/doi/10.1103/PhysRevD.104.103009#:~:text=%28FRBs%29,more%20evidence%20for%20the%20axion). In any case, these signals are likely difficult to detect with current detectors, but they are conceptually important: **a collapsing scalaron could produce a unique gravitational wave signature** distinct from, say, the merger of two black holes. It might be a burst or an “echo” following a primary collapse.

**Endpoint:** Ultimately, the end state of the collapse regime is a **black hole (with mass mostly from the scalaron) plus possibly a residual halo of scalaron material**. The black hole will obey the usual laws (e.g. if it’s non-rotating, it is a Schwarzschild BH characterized just by its mass and possibly charge). The scalaron field far away will revert to whatever the ambient conditions dictate (e.g. the halo outside the new BH might still have a solitonic structure if some scalaron remains bound outside). Over time, any scalar “hair” decays – calculations show that massive scalar perturbations around a BH have quasi-normal modes that damp out. However, one exciting aspect is that if the black hole is spinning, it can *regenerate* a scalar cloud via superradiant instabilities – essentially acting like a particle collider for the scalaron. That strays beyond our current scope, but it means **black holes could be laboratories for the scalaron**, leading to phenomena like gravitational atom spectra or persistent oscillating fields (with potential astrophysical signals).

**Transient vs Permanent Modified Gravity:** In this strong-field regime, does the scalaron produce any **modified gravity effect**? Potentially yes, but only transiently. During collapse, the scalar field’s stress-energy is complex and could cause departures from pure vacuum GR in the surrounding spacetime (a nontrivial energy-momentum distribution outside the horizon). This could be considered a **modified gravity regime** in the sense that the solution isn’t just Schwarzschild; there is a scalar field contributing to the metric. However, as the scalar dissipates or falls in, one returns to standard GR with a Schwarzschild (or Kerr) metric for the BH. So the modifications to gravity are not long-lived unless some mechanism stabilizes the scalar field outside (which typically would require either a special coupling or a non-decay into the BH; some models of scalar–tensor gravity do admit *hairy black holes*, but those often involve additional potentials or breaking of no-hair conditions).

In summary, the collapse regime shows the scalaron’s ability to behave like a **relativistic field** in strong gravity: it can form black holes just as normal matter can, but with accompanying phenomena of scalar wave emission and temporary hair. This regime underscores that the adaptive scalaron truly spans from **quantum to classical to relativistic gravity**: it is born as a quantum wave, lives as a classical halo, and dies (in a sense) as a contribution to a black hole. The formation of black holes also ties into entropy and information in ways we discuss next.

**Entropy, Decoherence, and the Arrow of Time**

An appealing aspect of the adaptive scalaron framework is that it offers a concrete picture of how *quantum cosmological initial conditions* evolve into *classical structure* with an associated increase in entropy – illuminating the **cosmological arrow of time**. Here we analyze the flow of entropy and role of decoherence as the scalaron progresses through its different regimes.

**Low-Entropy Beginnings:** At early times, or in large voids, the scalaron field can be approximated as a homogeneous, coherent state – essentially a zero-entropy configuration. In quantum terms, if the field is in a pure state (e.g. the BEC ground state), its entropy (von Neumann entropy of the density matrix) is minimal. Cosmologically, this aligns with the idea that the early Universe (nearly homogeneous density, including dark matter) had remarkably low entropy in its gravitational degrees of freedom​[preposterousuniverse.com](https://www.preposterousuniverse.com/blog/2007/12/03/arrow-of-time-faq/#:~:text=The%20observed%20macroscopic%20irreversibility%20is,the%20origin%20of%20the%20universe)​[preposterousuniverse.com](https://www.preposterousuniverse.com/blog/2007/12/03/arrow-of-time-faq/#:~:text=our%20current%20cosmological%20horizon%2C%20that%E2%80%99s,all%20of%20the%20matter%20in). All the scalaron quanta oscillating in phase is analogous to a system at zero temperature. This is consistent with the **Past Hypothesis** in cosmology: the Universe started in a special low-entropy state (smooth, no structure). Indeed, if the scalaron is an axion produced by misalignment, its initial condition is a spatially constant field value with some zero-velocity – a very ordered state.

**Structure Formation and Entropy Increase:** As time progresses, small perturbations in the scalaron grow under gravity (just like perturbations in CDM would). Overdense regions become halos, and within halos the field decoheres into multiple modes. **Every merger, virialization event, or interaction effectively increases the coarse-grained entropy of the scalaron field.** The formerly coherent phases become scrambled, corresponding to a move from a pure state to a mixed state (when only gross degrees of freedom are observed)​[en.wikipedia.org](https://en.wikipedia.org/wiki/Arrow_of_time#:~:text=match%20at%20L436%20decoherence%20is,In%20the%20language). In thermodynamic language, the system thermalizes gravitationally – though not in temperature, but in dynamical equilibrium. The **thermodynamic arrow of time** is manifest: entropy (disorder in phase space) increases as structures form. This is tightly connected with **decoherence**: quantum decoherence is essentially an entropy increase at the microscopic level​[en.wikipedia.org](https://en.wikipedia.org/wiki/Arrow_of_time#:~:text=match%20at%20L436%20decoherence%20is,In%20the%20language). When the scalaron’s wavefunction entangles with complicated environmental degrees (like multi-stream motions, baryonic clumps, etc.), the relative phases become effectively random. *“Decoherence is a form of increase in microscopic disorder – in short, decoherence increases entropy.”*​[en.wikipedia.org](https://en.wikipedia.org/wiki/Arrow_of_time#:~:text=match%20at%20L436%20decoherence%20is,In%20the%20language). Thus the growth of cosmic structures provides a concrete mechanism for decoherence: the many-body gravitational interaction plays the role of an environment that irreversibly correlates and scrambles the phases of the field. The process is unidirectional in time – once phases are lost, it’s practically impossible to spontaneously recohere the field into its original pure state (that would require an extraordinarily tuned time-reversal of all interactions, prohibited by the second law)​[en.wikipedia.org](https://en.wikipedia.org/wiki/Arrow_of_time#:~:text=decoherence%20is%20a%20form%20of,In%20the%20language).

**Arrow of Time:** The cosmological arrow of time (the direction in which the universe becomes more structured and more entropic) is intimately tied to gravity and thus to the behavior of dark matter. In our scenario, the scalaron starts in a low-entropy state and ends up contributing to extremely high entropy structures (black holes). Roger Penrose famously argued that gravity’s tendency to clump matter leads to the arrow of time, since a homogeneous gas has low gravitational entropy while a collection of black holes has maximal entropy​[preposterousuniverse.com](https://www.preposterousuniverse.com/blog/2007/12/03/arrow-of-time-faq/#:~:text=our%20current%20cosmological%20horizon%2C%20that%E2%80%99s,all%20of%20the%20matter%20in). Our adaptive scalaron provides a step-by-step realization: initial smooth scalar field (low entropy) -> halo with core and incoherent halo (higher entropy) -> possibly black hole formation (very high entropy). For example, consider the entropy in units of Boltzmann’s constant: A single 10^9 $M\_\odot$ soliton core has some entropy associated with its excitations, but if that core collapses into a $10^9 M\_\odot$ black hole, the entropy jumps enormously (a Schwarzschild BH of that mass has $S \sim 10^{90}$ in dimensionless units)​[preposterousuniverse.com](https://www.preposterousuniverse.com/blog/2007/12/03/arrow-of-time-faq/#:~:text=our%20current%20cosmological%20horizon%2C%20that%E2%80%99s,all%20of%20the%20matter%20in). This dwarfs any entropy previously in the dark matter distribution. The **entropy of the universe increases by orders of magnitude as scalaron cores collapse into black holes** – an irreversible step cementing the arrow of time.

**Entropy Flow in Phases:** We can delineate entropy flow through the scalaron’s phases: (i) *Quantum-coherent phase:* minimal entropy (all particles in one mode). (ii) *Decoherent halo phase:* entropy produced as modes populate and phases randomize. The dark matter (scalaron) effectively gains an entropy akin to phase-space volume occupied by its granules. (This can be quantified by an effective phase-space density decrease – the wave nature sets a max phase-space density, and as modes excite, the system “warms up” in a sense.) (iii) *Black hole phase:* a huge entropy increase concentrated in the BH horizon. Notably, most of that entropy is gravitational (in the geometry), not carried by the scalar field’s degrees of freedom anymore. One can think of it as the scalaron’s entropy being dumped into the spacetime itself (the BH). This might be relevant to the information problem: the pure state of many scalar particles turns into the mixed state of a Hawking radiation/BH horizon ensemble.

**Cosmological Arrow and Quantum Arrow:** It has been argued that the **thermodynamic arrow of time (entropy increase)** gives rise to the **quantum arrow (decoherence)**​[en.wikipedia.org](https://en.wikipedia.org/wiki/Arrow_of_time#:~:text=,6%20Quantum%20arrow%20of%20time)​[en.wikipedia.org](https://en.wikipedia.org/wiki/Arrow_of_time#:~:text=The%20conventional%20approach%20is%20to,dynamics%20is%20assumed%20to%20be). In our scenario, they are indeed aligned: as structures form and entropy increases, the scalaron decoheres. The arrow of time is evident – we see structure (and decoherence) growing in one time direction (forward). Time-reversing a virialized halo full of granular interference patterns would require them all to converge and phase-align into a smooth field – a fantastically improbable “entropy-decreasing” event.

In conclusion, the adaptive scalaron model is fully compatible with and illustrative of the cosmological arrow of time. It starts in an ordered state and naturally evolves into disordered states. The *decoherence of the scalaron field links microscopic quantum irreversibility to macroscopic gravitational irreversibility*. This not only provides a satisfying narrative (the universe transitions from quantum to classical in the dark sector as time goes on), but also has practical implications: once decohered on small scales, the scalaron can be treated classically, justifying why we can use classical N-body simulations for most structures. However, tiny residual quantum effects (like core oscillations) remain as relics of the low-entropy past. The entropy considerations additionally highlight that **black holes are the final depositories of the scalaron’s information**, pushing us to consider how (or if) any information about the field’s initial quantum state might be encoded or lost – an issue we revisit when discussing “wavefunction memory” in §7.

**Bridging Theoretical Domains: From Dark Matter to Geometry**

One of the compelling strengths of the adaptive scalaron hypothesis is how it serves as a bridge between disparate theoretical domains. Here we examine these connections in detail, showing that the scalaron provides a single framework that touches upon **dark matter phenomenology**, **modified gravity**, **quantum coherence vs. classicality**, and even **twistor/geometric formulations of gravity**. We discuss each in turn.

**Dark Matter Phenomenology and Small-Scale Structure**

The scalaron was originally motivated by dark matter problems, so it must successfully reproduce (or improve upon) the phenomenology of cold dark matter. We find that it indeed captures the large-scale successes of CDM while offering solutions to several **small-scale puzzles**:

* **Large-Scale Structure:** As noted, on scales of clusters and beyond, the scalaron behaves indistinguishably from standard CDM​[nature.com](https://www.nature.com/articles/nphys2996#:~:text=galaxies%2C%20with%20only%20one%20free,%E2%89%B2%2013%20in%20our%20simulations). It has the same linear growth and clustering properties (for appropriate initial conditions). Therefore, it matches observations like the cosmic microwave background (which constrains the matter power spectrum on large scales) and galaxy two-point correlations, etc. The introduction of a quantum Jeans scale (due to the scalar’s effective “quantum pressure”) only affects the very small-scale cutoff of the power spectrum​[nature.com](https://www.nature.com/articles/nphys2996#:~:text=The%20conventional%20cold,achieve%20cosmological%20simulations%20of%20this). For $m \sim 10^{-22}$ eV, structure formation is **delayed on very small scales**: no halos below ~$10^7$–$10^8 M\_\odot$ (since perturbations below a kpc scale are wiped out by the wave support)​[nature.com](https://www.nature.com/articles/nphys2996#:~:text=The%20conventional%20cold,achieve%20cosmological%20simulations%20of%20this). This is actually desirable because CDM arguably over-predicts low-mass halos.
* **Cusp–Core Problem:** Many dwarf galaxies and low-surface-brightness galaxies exhibit shallow density cores instead of the steep cusps predicted by CDM. The scalaron’s fuzzy wave regime naturally yields **kiloparsec-scale cores** in dwarf halos​[nature.com](https://www.nature.com/articles/nphys2996#:~:text=The%20conventional%20cold,achieve%20cosmological%20simulations%20of%20this). The solitonic core density profiles match the observed cores in many cases (with core sizes scaling inversely with halo mass, qualitatively similar to trends needed to explain bigger cores in smaller galaxies). For example, a typical $10^{10} M\_\odot$ halo might get a $\sim1$ kpc core of the right density to fit dwarf spheroidal kinematics. This happens *without fine-tuning*, emerging from the physics of the scalar field.
* **Missing Satellites Problem:** CDM predicts a wealth of subhalos around Milky Way-like galaxies, far more than the number of observed dwarf satellite galaxies. The scalaron addresses this in two ways: (1) The initial power spectrum cutoff means fewer small halos form in the first place (similar to warm dark matter). (2) Even those that form can be more easily tidally destroyed (see next point), further reducing the count. Thus, the **abundance of satellite halos is naturally suppressed**. Simulations of fuzzy dark matter show that for $m \approx 10^{-22}$ eV, the halo mass function is significantly reduced below $\sim10^8 M\_\odot$, bringing the number of subhalos in line with observed satellites for a Milky Way-sized host, without conflicting with larger-scale structure​[nature.com](https://www.nature.com/articles/nphys2996#:~:text=The%20conventional%20cold,achieve%20cosmological%20simulations%20of%20this).
* **Tidal Stability and Dynamical Friction:** Because subhalos of scalaron dark matter have lower concentrations (especially if their cores are big relative to their size) and because they can undergo ram-pressure-like wave stripping, they may disrupt more easily when orbiting a larger host. The study by Du *et al.* found that a solitonic core under tidal stress will undergo a **runaway disruption once it loses enough mass**, dissolving faster than a CDM subhalo would​[arxiv.org](https://arxiv.org/abs/1801.04864#:~:text=of%20tidal%20disruption%2C%20highlighting%20the,the%20minimum%20mass%20of%20cores). This implies that observed stellar streams might show fewer perturbations (gaps) from subhalos, consistent with some current stream observations that hint at a lack of heavy perturbers. Additionally, fuzzy subhalos impart less dynamical friction (their cores fluffier), possibly alleviating issues like the “too big to fail” problem (the most massive CDM subhalos predicted seem to not host visible dwarfs – in scalaron DM those subhalos might have been erased or never formed).
* **Galaxy Formation Timing:** With small-scale power suppressed, galaxy formation, particularly for tiny protogalaxies, is delayed. Simulations indicate galaxy formation in a fuzzy DM universe starts later (no objects before $z\sim 10$–$20$ for $m\sim10^{-22}$ eV)​[nature.com](https://www.nature.com/articles/nphys2996#:~:text=allow%20us%20to%20determine%20Image,%E2%89%B2%2013%20in%20our%20simulations). This could be related to recent JWST observations of early galaxies: if JWST had found an *excess* of dwarf galaxies at high redshift, it would challenge fuzzy DM. Conversely, if it finds fewer small galaxies than CDM expects, that supports a scalaron-like DM. This is a test in progress.

In summary, the scalaron as dark matter **preserves large-scale concordance** (like CDM) but **mitigates small-scale conflicts** by virtue of its finite quantum pressure and wave dynamics. The presence of a consistent quantum core–halo structure across halos is a clear prediction: e.g., a Milky Way-sized halo should harbor a $\sim 100$ pc dense soliton (perhaps affected by the central BH)​[sciencedirect.com](https://www.sciencedirect.com/science/article/abs/pii/S2212686419303334#:~:text=Dynamical%20evidence%20of%20a%20dark,is%20smaller%2C%20pc%2C%20of%20mass), and ultra-faint dwarf galaxies should be supported by 0.5–1 kpc cores (leading to cored velocity profiles). These phenomenological successes are a strong motivator for the scalaron theory, and future observations (e.g. precision stellar kinematics in dwarfs, strong gravitational lensing for halo profiles) can further test these predictions.

**Connections to Modified Gravity (Scalar-Tensor Theories and Screening)**

While the scalaron mostly behaves as dark matter, in certain regimes it can mimic modified gravity by contributing an additional force or altering the effective gravitational constant. Here we connect the scalaron to known frameworks of modified gravity:

* **Scalar-Tensor Equivalence:** It is well known that many modified gravity theories can be cast as General Relativity plus a scalar field. For example, **$f(R)$ gravity** (where the Lagrangian is a function of Ricci scalar) can be rewritten as GR with an extra scalaron field $\phi = f'(R)$ that mediates a force​[inspirehep.net](https://inspirehep.net/files/2e446914ecf6a036ff82d5de6c35a1dd#:~:text=that%20power,%282015%29%20used). In our case, we introduced the scalaron as a physical field from the outset, but one could imagine that what we call $\phi$ might partly originate from such an $f(R)$ term or similar. Indeed, the *original scalaron* (Starobinsky’s) was massive ($10^{-12}$ GeV) and drove inflation​[inspirehep.net](https://inspirehep.net/files/2e446914ecf6a036ff82d5de6c35a1dd#:~:text=that%20power,%282015%29%20used). Our scalaron is ultralight ($10^{-22}$ eV) and survives to late times as DM. But both are scalar degrees of freedom coupled to curvature. We might extend the action to $f(R,\phi)$ or include a nonminimal coupling $\xi \phi^2 R$ to directly tie $\phi$ to the Ricci scalar. An open question (see §7) is the best coupling form, but qualitatively, the scalaron can act as a **mediator of an extra gravitational interaction** on scales where it is light.
* **Chameleon Screening:** The **chameleon mechanism**​[aanda.org](https://www.aanda.org/articles/aa/pdf/2014/01/aa21061-13.pdf#:~:text=Aims,its%20context%20and%20applied%20it) is a way that a scalar field can evade detection in high-density environments by becoming massive. Our adaptive scalaron is essentially a *cosmological chameleon*. In galaxies (which are relatively dense compared to voids), the scalaron’s amplitude may decrease and its effective mass increase, thereby diluting any “fifth force” it would otherwise produce. The result is that in the solar system or Earth laboratory, the scalaron’s effects (if any) are hidden, consistent with no deviation from Newton/Einstien observed. But in intergalactic space or galaxy outskirts, the field is light and could mediate a force. This is analogous to how chameleon dark energy fields behave – but here the field also carries the dark matter energy density. The work of Salzano *et al.* (2014) directly explored a scalar field with **mass and coupling that scale with local properties** (inspired by chameleon, symmetron, etc.) and found it can fit galaxy dynamics as an alternative to dark matter​[aanda.org](https://www.aanda.org/articles/aa/pdf/2014/01/aa21061-13.pdf#:~:text=Aims,its%20context%20and%20applied%20it)​[aanda.org](https://www.aanda.org/articles/aa/pdf/2014/01/aa21061-13.pdf#:~:text=Results,of%20galaxies%2C%20show%20evident%20correlations). In our scenario we have both: the scalar provides the mass (dark matter) *and* an extra attraction (modified gravity) in varying proportions.
* **MOND-like Behavior:** One might wonder if the scalaron can reproduce the phenomenology of MOdified Newtonian Dynamics (MOND) in galaxies – the empirical scaling where $g\_{\rm obs} \approx \sqrt{g\_{\rm bar} a\_0}$ at low accelerations. Pure fuzzy dark matter generally does not yield MOND-like behavior; it still requires the presence of dark mass to explain rotation curves. However, in some regimes, a scalar-mediated force on baryons could effectively augment gravity. For instance, in Khoury’s superfluid DM, the condensate’s phonons create a $1/r$ force that adds to Newtonian gravity to give MOND’s form​[scipost.org](https://scipost.org/submissions/2109.10928v1/#:~:text=the%20last%20few%20years,studied%20physics%20of%20superfluidity). In the adaptive scalaron context, one could speculate that if a galaxy’s halo is partially in a condensed state, collective excitations might couple to baryons. This is beyond our base model (which has $\phi$ only gravitationally coupled), but if $\phi$ coupled to baryons with a strength $\beta$ (like a scalar–matter coupling), it would introduce a fifth force $F\_\phi = \beta \nabla \phi$. If that force becomes significant in the outer parts of galaxies (where $\phi$ might still be ambient and not too heavy), it could mimic MOND. We won’t derive this here, but simply note that *different effective theories can emerge from the scalaron in different limits*. In one limit (many quanta, decohered), it’s CDM; in another (condensed, excited states), it might approximate a scalar-tensor modified gravity law. The richness of scalar field models allows, in principle, a single framework to produce both phenomena.
* **Relativistic Effects and $f(R,T)$ Gravity:** Some modified gravity theories also include dependencies on the stress-energy trace $T$ (as in $f(R,T)$ gravity) or other invariants. The adaptive scalaron could be extended similarly: its effective potential or mass might depend on $T$ (local matter density). This would explicitly implement the adaptive behavior: $m\_\text{eff}^2(\phi) = m^2 + \alpha T$ for example, where $T$ large (high density) gives a large $m\_\text{eff}$. The result is like a built-in screening. This approach is in line with modern scalar-tensor models that ensure they pass solar system tests by design. In our context, it’s a way to parametrize “the scalaron knows about its environment.”
* **Gravitational Constant Variations:** If the scalaron has a nonminimal coupling, it can cause the *effective gravitational constant* $G\_{\rm eff}$ to vary. For example, in Brans-Dicke theory (scalar-tensor theory), $1/G\_{\rm eff}$ is replaced by a scalar field that can vary in space and time. An ultralight scalaron could cause subtle variations: perhaps between voids and clusters, or from early to late times. Current observations (like primordial nucleosynthesis, CMB, etc.) strongly constrain any such variations, so the scalaron coupling must likely be small. But it’s an avenue where cosmology and local gravity tests cross-constrain the model.

Overall, the adaptive scalaron sits naturally in the scalar-tensor modified gravity landscape. It can be viewed as a *unifying field that carries matter (dark matter) and mediates modified gravity as needed*. It embodies the philosophy that instead of separate dark matter particles and a modification to gravity, a single scalar field’s behavior changes with scale: in cosmic voids or low-acceleration regions it effectively strengthens gravity (since it’s a source of extra attraction distributed more smoothly than clumpy DM), and in high-density regions it behaves just like dark matter (clustered mass that simply contributes to gravity in the usual way). This unity is a significant conceptual simplification, though realizing it quantitatively requires careful model-building to satisfy all constraints.

**Quantum Coherence, Decoherence, and Classical Transition**

The scalaron provides a concrete case study in the quantum-to-classical transition in cosmology. It literally *has its foot in both worlds*: a quantum wave on large scales and a classical particle ensemble on small scales. This duality is typically hard to address in cosmological structure formation, but here it is front and center.

* **Quantum Origin of Structure:** In inflation theory, quantum fluctuations are the seeds of cosmic structure. Likewise, in our model the initial fluctuations of the scalaron field (perhaps generated during inflation or during its misalignment production) are quantum in nature. However, once inflation ends and the universe expands, these perturbations are often treated classically (as in standard perturbation theory). The scalaron’s wave description validates this for large scales while retaining a *literal quantum treatment* for small scales when needed. Essentially, it says: we don’t need to artificially add random phases to mimic classical perturbations – the theory itself will decohere and behave classically at the appropriate point, due to many-body interactions.
* **Decoherence in the Dark Sector:** Usually, when we think of quantum decoherence, we consider particles interacting with photons or a heat bath. Here, remarkably, **gravity itself (and self-interaction of the field) provides the decohering “bath.”** The scalaron in a rich environment (galaxy) acts like a particle system because its wavefunction has effectively collapsed (not via an observer, but via chaotic dynamics). This is a more objective notion of wavefunction collapse – environment-induced decoherence without measurement​[en.wikipedia.org](https://en.wikipedia.org/wiki/Arrow_of_time#:~:text=match%20at%20L436%20decoherence%20is,In%20the%20language). In principle, one could compute a density matrix for the scalaron field, integrate out small-scale modes, and see that the off-diagonal terms (coherences) vanish over time, leaving a diagonal mixture corresponding to various classical field configurations (which is how an N-body system would appear).
* **Re-coherence and Reversibility:** Once decohered, could the scalaron ever re-cohere? In normal circumstances, no – that would require entropy decrease. For example, two decoherent halos merging won’t produce a coherent larger halo; instead, they’ll make an even more chaotic one. Only in very special situations (perhaps at the centers of halos, during a very slow accretion without disturbance) can coherence be partially preserved (hence the persistence of soliton cores). Another interesting thought: If two identical solitonic cores merge head-on with precisely the right phase difference, they might interfere and form a single larger soliton (coherent merger). Simulations indicate solitons can merge and settle into a bigger soliton if conditions are ideal. This is like two Bose condensates merging – if phases align, one gets a condensate; if not, there could be waves and turbulence (decoherence). Thus, **phase alignment matters**. Structure formation is random enough that widespread phase alignment is rare, so typically coherence is local (within each core separately). The arrow of time argument suggests the universe favored the low entropy route initially (one phase everywhere) but as different regions went out of causal contact or collapsed at different times, their phases became uncorrelated.
* **Macroscopic Quantum Effects:** One might ask: are there any observable quantum effects remaining on large scales? Possibly subtle ones. For instance, the fluctuations (granules) in halos are essentially a macroscopic quantum interference pattern. They might cause *random walks* of stars or gas (as the potential fluctuates). This could in principle be distinguished from classical shot-noise fluctuations. Another effect: the very existence of stable solitonic cores is a macroscopic quantum phenomenon (no classical particle collection would remain so condensed without thermal pressure or anisotropy causing collapse or cusp formation). So observing a stable dark matter core in a small galaxy is effectively observing a quantum effect on kiloparsec scales.
* **Connection to Many-Worlds and Measurement:** Philosophically, one could connect this to interpretations of quantum mechanics. In Many-Worlds terms, as the scalaron decoheres, the “branching” of the wavefunction could correspond to different classical halo configurations. But since we only observe one classical outcome, it’s as if the wavefunction’s possibilities have branched into different classical histories – one of which we follow. This is analogous to measurement, but happening in the cosmological context. While deep interpretation is beyond our scope, it’s fascinating that **cosmic structure formation could be viewed as a continual measurement-like process on the dark matter field**, collapsing it into classical structures.

In summary, the scalaron framework doesn’t just bridge quantum and classical physics conceptually; it demands we treat them in one unified description. It stands as a concrete model for how the classical universe we see (galaxies, clusters, black holes) could emerge from an underlying quantum wave reality, with decoherence providing the link – and it does so in a way that is quantitatively testable via astrophysical phenomena like those outlined in §6.

**Twistor Theory and Geometric/Topological Encoding**

Finally, we turn to a more speculative but profound connection: the relation of the scalaron field to **twistor theory and topological structures** in spacetime. Twistor theory, originated by Penrose, seeks to recast physics in a framework where fundamental objects are described in a complex projective space (twistor space) rather than spacetime, potentially uniting quantum theory and gravity​[pos.sissa.it](https://pos.sissa.it/323/003/pdf#:~:text=Broadly%20speaking%2C%20twistor%20theory%20is,as%20some%20of%20its%20historic). One might ask: *what does an adaptive scalaron look like in twistor space?* And can twistor ideas help encode the field’s “memory” – the information carried through its quantum-to-classical evolution?

* **Twistor Representation:** In twistor theory, solutions of field equations in spacetime can be encoded as geometric data (like holomorphic curves) in twistor space​[pos.sissa.it](https://pos.sissa.it/323/003/pdf#:~:text=Broadly%20speaking%2C%20twistor%20theory%20is,as%20some%20of%20its%20historic). For example, certain gravitational or Yang-Mills solutions correspond to algebraic curves in twistor space. A scalar field such as $\phi(x)$, especially if massless or light, could similarly be represented in twistor space (massless scalar fields correspond to certain cohomology classes in twistor space, whereas a massive scalar might require an extension of the twistor formalism). The *non-local relationship* between spacetime and twistor space​[pos.sissa.it](https://pos.sissa.it/323/003/pdf#:~:text=Broadly%20speaking%2C%20twistor%20theory%20is,as%20some%20of%20its%20historic) suggests that some global properties of the scalaron (like phase angles winding around large-scale structures, or topological phase defects if any) could be easier to analyze in twistor space.
* **Phase and Topology:** The scalaron’s phase pattern in space can have topological features. For instance, interference fringes and solitons can be associated with phase discontinuities. If the scalaron field ever had vortices or domain-like structures (not typically in fuzzy DM, but conceivable with certain initial conditions or multiple fields), those would be topologically nontrivial. Twistor theory, being adept at encoding topological and analytic properties of fields, might encode a **global phase configuration as a holomorphic object**. One could imagine that even if the scalaron loses local coherence, some imprint of its initial coherent phase distribution might be stored in a global twistor function (since twistor space can sometimes capture information that is obscured after decoherence in spacetime).
* **Memory of the Wavefunction:** When the scalaron collapses into a black hole, we encounter the question of information loss. Is the information in the scalaron field (the detailed phase-space configuration) lost to the singularity, or is it somehow encoded in subtle correlations (like Hawking radiation correlations or quantum gravity states)? While a full answer requires quantum gravity, twistor theory might provide a language to discuss what is preserved. For example, **gravitational wave memory** is an effect where after gravitational waves pass, detectors remain displaced – the spacetime has a “memory.” By analogy, a scalar field collapse might leave a memory in the surrounding spacetime (perhaps a permanent offset in the scalar field at infinity or some gravitational imprint). Twistor methods, which naturally handle radiation and infinity (via Penrose’s conformal compactification and null infinity structures), could be useful to formalize any such memory. If one were to compute the twistor space description of a collapsing scalaron configuration, one could potentially track what part of the twistor data goes “inside” (lost) versus what remains accessible at infinity.
* **Unification of Descriptions:** One dream is that twistor theory could unify the quantum field aspects and gravitational aspects of the scalaron in one geometric picture​[pos.sissa.it](https://pos.sissa.it/323/003/pdf#:~:text=Broadly%20speaking%2C%20twistor%20theory%20is,as%20some%20of%20its%20historic). In twistor space, a solution that at one limit looks like a linear wave and at another limit like a nonlinear gravitational shock might be represented continuously. For instance, perhaps the transition from a diffuse scalar field to a black hole corresponds to a deformation of a certain twistor space contour. This is highly speculative, but aligns with Penrose’s view of twistor theory as a path to quantum gravity​[royalsocietypublishing.org](https://royalsocietypublishing.org/doi/10.1098/rspa.2017.0530#:~:text=Twistor%20theory%20at%20fifty%3A%20from,general%20relativity%20and%20quantum%20mechanics). As the scalaron is a prime example of a quantum field that significantly affects geometry, it’s a natural playground for such ideas.
* **Twistor and Adaptive Field:** If we were to push the analogy, one might consider the scalaron as an *order parameter* in a twistor-based state. Topologically, the adaptive scalaron may ensure that certain invariants (like winding numbers or indices) remain unchanged even as it transitions. This could address “memory encoding”: maybe the field’s initial conditions are not entirely erased but partly stored in global quantities (like integrals of motion). Twistor space might make those integrals manifest. For example, the integrals of $\phi$ over large 3-volumes, or certain helicity components, might correspond to conserved charges in twistor space.

These ideas are admittedly on the frontier between established theory and conjecture. However, including them underlines the **comprehensiveness of the adaptive scalaron’s reach**: it encourages dialogues between cosmology and more abstract mathematics. Even if twistor methods are not yet fully applied to this scenario, the mere fact we can discuss them indicates the depth of the scalaron concept.

In conclusion of this section, we see that the adaptive scalaron is not an *ad hoc* construct but rather sits at a nexus of multiple frameworks. It behaves like dark matter yet has features of a modified gravity scalar; it originates from quantum physics but explains classical structures; and it even invites consideration in advanced formalisms like twistor theory. This unity is a strong sign that we may be capturing a piece of fundamental truth – that **a single field can weave together the quantum microphysics and the geometric macrophysics of our universe**.

**Observational Signatures and Testable Predictions**

A theory that unifies concepts must ultimately face the test of observation. The adaptive scalaron model makes a variety of **predictions**, many of which are distinguishable from standard $\Lambda$CDM or other new physics. We outline key observational targets, each corresponding to one of the scalaron’s characteristic behaviors:

* **Soliton Core Oscillations and Dynamics:** One striking prediction is the presence of **solitonic cores** in dark matter halos that can **oscillate** and **wander** slightly. In a galaxy with no central black hole, the scalaron core (of order hundreds of parsecs in dwarfs, smaller in bigger halos) should exhibit small-amplitude pulsations (density oscillating by a few percent) and a random walk about the halo center​[osti.gov](https://www.osti.gov/biblio/1851580#:~:text=Previous%20investigations%20found%20that%20the,of%20the%20excited%20states%2C%20the). These might be detectable via their gravitational influence on stars or gas. For example, star motions at the center of a dwarf galaxy might show an additional random velocity dispersion or an oscillatory component not explainable by stellar processes. In the Milky Way, if our inner halo hosts a scalaron core (likely truncated by the central BH’s gravity), it might cause subtle perturbations in the orbits of stars near the galactic center or possibly contribute to the observed Central Molecular Zone kinematics. Observationally, one could search for a **periodic change in the gravitational potential** of a dwarf galaxy core (maybe in systems like Eridanus II which might host a large dark matter core). Precision stellar stream or pulsar timing in the core region could also probe this. If we ever observe multiple snapshots of a dark matter-dominated core (e.g. through strong gravitational lensing over time), we might witness the core’s oscillation. Additionally, the **random walk of the core** could heat the central star cluster of a dwarf galaxy (stars gain energy when the dark potential moves around). This has been suggested as a way to test fuzzy dark matter: look at star clusters in dwarf galaxies for signs of dark matter-induced heating​[osti.gov](https://www.osti.gov/biblio/1851580#:~:text=,deducing%20constraints%20from%20stellar%20heating).
* **Tidal Disruption and Halo Substructure Patterns:** Because scalaron subhalos are less resilient, the pattern of **tidal streams and satellite disruptions** in our galaxy should differ from CDM predictions. In CDM, long thin stellar streams (like GD-1 or the Pal 5 stream) are expected to be punctured and perturbed by numerous subhalo flybys, creating gaps or wiggling the stream. The scalaron model predicts **fewer and gentler perturbations**. Instead of dozens of heavy subhalos carving the stream, we might see only the effect of one or two larger perturbers (the ones that survived) plus a more diffuse, wave-like perturbation from the granules of the dark matter halo. There is tentative evidence that observed streams *do not* show as many small-scale perturbations as a rich CDM subhalo population would produce. This absence is consistent with either a lack of subhalos (as fuzzy DM provides) or subhalos that are too weak to perturb (also the case here). One could quantify this by analyzing stream density power spectra: the adaptive scalaron model predicts a suppression of power on scales corresponding to subhalo masses below $\sim10^8 M\_\odot$, but perhaps an enhanced *continuous* perturbation from interference patterns on larger scales (kpc). Another signature is **tidal dwarf galaxies or orphan cores**: if a subhalo’s outer part is stripped, a lone soliton core might remain as a free-floating object. These would be difficult to detect, but perhaps through gravitational lensing or as microhalo perturbations of cold stellar streams. The existence of an unusually dense, small substructure with no extended halo would be a smoking gun for a soliton remnant. On cluster scales, one might look at **distribution of substructure** via strong lensing: fuzzy/scalaron DM yields fewer small mass clumps, so lensing anomalies (like flux ratio anomalies in lensed quasars) would be less frequent than in CDM. Current lensing constraints are starting to push into this regime, and a confirmed lack of small clumps (or core sizes in clumps) could favor the scalaron idea.
* **Scalar Wave “Echoes” and Gravitational Waves from Collapse:** As discussed, if a scalaron core collapses into a black hole, it can emit a burst of radiation. While direct detection of scalar waves is infeasible (they interact very weakly with detectors unless they mix with photons), gravitational waves could be observable. The model predicts **high-frequency gravitational wave bursts** from such events​[link.aps.org](https://link.aps.org/doi/10.1103/PhysRevD.104.103009#:~:text=%28FRBs%29,more%20evidence%20for%20the%20axion). These could occur for example at the moment a supermassive black hole forms at a galactic center (different from standard stellar collapse; this is dark matter collapse). The frequency could be extremely high (kHz or more), which is outside the band of LIGO but potentially in range of future detectors or resonant bar detectors. One might also search for an **astrophysical signature** of the collapse: e.g., if scalar radiation is emitted, does it couple to any standard model particles? If the scalaron is an axion, it could resonantly convert to photons in magnetic fields, possibly leading to a radio or X-ray flash. Some speculative connections even suggest axion mini-clump collapses might explain fast radio bursts (FRBs) by photon coupling​[link.aps.org](https://link.aps.org/doi/10.1103/PhysRevD.104.103009#:~:text=The%20axion%20objects%20such%20as,secondary%20gravitational%20wave%20production%20associated) – though that requires special conditions. Gravitational wave **“echoes”** are another idea: if after a black hole merger there’s a remnant scalar cloud or exotic structure, one could see repeated echo-like signals in the ringdown. Observers have looked for echoes in LIGO data (none confirmed yet), but the scalaron model provides another context: a newly formed BH (from scalar collapse or merger) could have a quasi-bound scalar wave that leaks out, generating a series of diminishing “echo” pulses in GWs. Detecting such a pattern would hint at physics beyond the vacuum Kerr BH paradigm, possibly supporting the idea of scalar field remnants.
* **Black Hole Scalar Hair and Deviations:** Although a fully formed isolated BH will lose scalar hair, **during formation and in equilibrium with scalar surroundings, there could be observable effects**. For instance, the presence of a scalar field around Sgr A\* (the Milky Way’s BH) could be tested by precision measurements of star orbits (the scalar field’s extra mass or fifth-force could cause anomalous precession). So far, general relativity with a point mass fits S2’s orbit well, which constrains any extended mass (like a fuzzy core) to be quite small. This is consistent with the scalaron expectation that in a massive galaxy, the soliton core radius might be only tens of pc or less, and with Sgr A\*’s $4\times10^6 M\_\odot$ BH present, the core might be largely accreted or disturbed. Still, future telescopes (like a pulsar timing array using a pulsar orbiting Sgr A\*) could detect even small deviations. **Testing no-hair theorem**: Some modified gravity or dark matter models allow black holes with hair. If, say, a stable scalaron configuration (a boson star) coexists or oscillates around a BH, one might detect deviations in the gravitational potential at horizon scales – for instance, the shape of the black hole shadow in EHT observations could differ. At present, M87\*’s shadow was consistent with GR, limiting any extensive scalar structure. But these are still crude tests. Another angle: **binary black hole mergers**. If dark matter around binaries is significant, the merger waveforms could be affected by dynamical friction or scalar radiation. LIGO/Virgo waveforms so far show no clear need for additional effects, implying that by the time of merger, either the scalaron around each BH was negligible or its effects are below detection. Still, a future space-based GW observatory (LISA) observing massive BH mergers might see disparities that hint at a scalar field influence (like a differing energy loss).
* **Halo Density and Structure Patterns:** The core–halo relation in fuzzy scalaron DM is very specific: core mass and halo virial properties are linked (e.g., $M\_{\rm core} \propto (M\_{\rm halo}/10^9M\_\odot)^{1/3}$ roughly, from simulations). Observationally, this means there should be a correlation between the inner dark matter density of galaxies and their outer halo mass. Some data on dwarf galaxies shows a trend of core size with halo mass consistent with FDM expectations, but it’s still debated. If future surveys map many dwarf galaxy rotation curves and get reliable halo masses (perhaps via outer halo tracers or lensing), they could confirm or refute this relation. **A soliton–halo relation** is a unique prediction​[osti.gov](https://www.osti.gov/biblio/1851580#:~:text=We%20report%20that%20a%20fuzzy,As%20the); CDM has no such fixed relation (core properties vary widely depending on baryonic feedback, etc.). Additionally, the **absence of cuspy halos at low mass** and the **flattening of the concentration–mass relation** below a certain scale are predictions we can test with upcoming telescopes (JWST finding dwarf halos at high $z$, or 21-cm surveys mapping small halo gas).
* **Direct Detection and Oscillations:** If the scalaron is an axion, it has a very low mass, meaning its field oscillates at a frequency $m \sim 10^{-22}$ eV $\approx 3\times10^{-8}$ Hz (period ~1 year). This homogeneous oscillation could, in principle, produce a tiny periodic variation in gravitational potentials or constants. Some have proposed searching for axion DM via precision atomic clocks or resonant masses looking for an oscillating signal at the mass frequency. Given our scalaron has such a low frequency, this is challenging (year-long periods, tiny amplitude). But concepts like atom interferometers in space might someday detect a coherently oscillating cosmic field. If detected, that would be a direct evidence of the scalaron (and would measure $m$ outright). While highly futuristic, it’s good to note that the scalaron DM could be probed in labs if its coupling to standard model exists (like the axion-photon coupling, which experiments such as ADMX and others try to exploit, though those are tuned to higher frequencies for heavier axions).

To summarize, the adaptive scalaron framework offers a rich menu of observational tests: from **core dynamics in dwarf galaxies** and **satellite stream perturbations**, to **gravitational wave bursts** and **exotic black hole effects**. Many of these are within reach of current or near-future facilities. A positive detection of any (e.g., discovery of a dark matter soliton through stellar kinematics, or signs of suppressed substructure, etc.) would bolster the case for this theory. Conversely, if observations continue to match CDM on all scales (including small scales), it will tightly constrain the allowed parameter space of an ultralight scalaron (as already, Lyman-$\alpha$ forest data suggests $m \gtrsim 10^{-21}$ eV, or else too much small-scale suppression). Thus, RFT 9.0 not only provides theoretical consistency but also yields **concrete predictions that can falsify or verify the scalaron hypothesis** in the coming years.

**Open Questions and Future Directions**

While the adaptive scalaron model is comprehensive, it also raises many questions and avenues for further research. We highlight several open theoretical questions and goals for future simulations and studies:

* **Scalaron–Curvature Coupling (f(R, φ)) Structure:** What is the most natural and self-consistent way to include the scalaron in the gravitational action? We introduced it minimally, but one could have an action $S = \int d^4x \sqrt{-g},f(R,\phi)$ that directly ties $\phi$ to curvature. A specific form might reproduce the adaptive behavior (for instance, $f(R,\phi) = R + \alpha \phi^2 R - m^2 \phi^2 - \lambda \phi^4 ...$). The open question is: *can we derive the effective environmental dependence of $\phi$’s mass or coupling from a fundamental $f(R,\phi)$ form?* This might require nonlinear analysis of field equations to see how local curvature (or stress-energy) influences the scalaron mass. Work in scalar-tensor theory could guide this, but our scenario is complicated by the need for ultralight mass and cosmic coherence. Another sub-question: how does the scalaron interact with the trace of the stress tensor $T$? In an $f(R,T)$ theory, one could incorporate matter effects explicitly. The goal would be a unified Lagrangian that in the Einstein frame yields a potential for $\phi$ that automatically has a density-dependent minimum (like chameleon). Solving this could make the theory more predictive (fewer free functions to hand-tune the adaptation).
* **Scalaron and Torsion / Emergent Gravity:** Could the scalaron be related to spacetime torsion or an emergent metric phenomenon? In Einstein–Cartan theory (a GR extension including torsion), a scalar field can induce a torsion field or be influenced by it​[sciencedirect.com](https://www.sciencedirect.com/science/article/abs/pii/S000349161830099X#:~:text=Einstein%20gravity%20with%20torsion%20induced,We). Some modern approaches, like teleparallel gravity, allow a **scalar-torsion** coupling (where gravity is encoded in torsion instead of curvature, and a scalar interacts with that)​[link.aps.org](https://link.aps.org/doi/10.1103/PhysRevD.101.024017#:~:text=Post,curvature%20gravity). Exploring a **$f(T, \phi)$** theory (teleparallel equivalent of $f(R, \phi)$)​[sciencedirect.com](https://www.sciencedirect.com/science/article/pii/S0370269324005264#:~:text=gravity%20www,is%20the%20canonical%20scalar%20field) might provide alternative insights – for instance, torsion could screen the scalar in dense regions differently. Also, emergent gravity ideas (like Verlinde’s) suggest that what we call “dark matter” might emerge from microscopic dof. The scalaron might be a more concrete realization: perhaps an underlying topological field in a higher-dimensional or quantum gravity theory that *emerges* as both geometry and matter in 4D. This is speculative, but future theoretical work could try embedding the scalaron in string theory or holography. Is there a dual description where the scalaron’s phases correspond to something like topological braids or twistors? If so, it could connect to quantum error-correcting codes or other modern quantum gravity concepts where “memory” is stored in nonlocal correlations. **Emergent metric models**: Another path is to consider if the metric itself could be an emergent composite of the scalaron (and perhaps other fields). For example, some bi-metric theories or condensate theories (like superfluid vacuum theory) conceive that spacetime might be a condensate of underlying fields. The scalaron as a cosmic BEC could hint that spacetime’s properties (like inertia or the speed of light in vacuum) might slightly change in regions of different scalaron density. No evidence of that exists, but it’s a theoretical curiosity – an environment-dependent scalar field could lead to environment-dependent effective metric for matter (like varying $c$ or $G$ in extreme cases). Exploring consistency of such ideas with precision tests would be needed.
* **Scalaron Collapse and Wavefunction Memory Encoding:** We touched on the fate of information in scalaron collapse. This remains an open problem: *does the scalaron’s wavefunction leave any imprint after forming a black hole?* Hawking’s semiclassical argument suggests the information is lost (or at least not visible in Hawking radiation, leading to the information paradox). In quantum gravity, likely information is preserved in correlations. If the scalaron is fundamental, then presumably unitarity applies to it as well – meaning the full quantum state evolution (scalaron + gravity) is unitary. How is the initial pure state of a soliton encoded in the outgoing Hawking radiation or the final BH state? One speculation: because the scalaron was a coherent state, maybe the black hole’s quantum state reflects that (e.g., a specific phase space distribution of microstates). Is there perhaps a subtle way that the *global phase or some quantum number of the scalaron survives*? For instance, if the scalaron had a global U(1) (like particle number), then black hole no-hair would mean it all went in. But maybe cosmic scalaron fields don’t have a strictly conserved number (axion number is broken by mass term). Still, baryon number sometimes is considered to possibly leave imprint (like a BH might carry a gravitational coupling to global charges). For future work: simulate a scalar field collapse in full GR (numerically) and track entropy of the field vs. area increase of BH – do we see unitary evolution? Also, could **gravitational memory** (permanent metric changes from wave emission) encode some of the information? This ties to our twistor discussion – maybe the outgoing gravitational wave carries a memory that is mathematically related to the initial scalar configuration. Unlocking this could provide a toy model for resolving information paradox in a simplified setting.
* **Phase Maps and Dynamic Transitions:** We currently qualitatively describe “phases” (quantum vs classical) – can we make a **phase diagram** for the scalaron? For example, axes could be local mass density vs. scalaron de Broglie wavelength (or velocity dispersion), and one could delineate regions: “coherent BEC”, “incoherent wave soup”, “classical collisionless”, “unstable collapse”. Having such a diagram would help identify in a given astrophysical context (e.g. at radius X in a halo of mass Y, or at redshift Z) which regime applies. To make it precise, one might define an order parameter for coherence (like the fraction of mass in the ground state mode). Simulations can measure this fraction in halos of various masses and redshifts, to empirically map the transition. Likewise, dynamic processes: how does a region transition from one regime to another? For example, as a dwarf falls into a cluster, its scalaron core might feel increasing tidal field – does it gradually decohere more (exciting more modes), or perhaps if stripped down to just a core, does it *re-cohere* in isolation? Investigating **time-dependent transitions** (through controlled simulation experiments where environment density is changed) would shed light on the hysteresis or irreversibility of these phase changes. Another open point is whether **mixed models** (part scalaron, part ordinary CDM) produce new phenomena – e.g., a fraction of dark matter as scalaron could still form a small soliton in the center and thus give cores, while CDM dominates outer halo. Does this mix solve issues that pure fuzzy DM might have with, say, galaxy counts? Studies of mixed FDM+CDM show intermediate outcomes​file-pbs5tcrmsvz7ndprsed51h; it would be interesting to see if RFT cosmology can accommodate multi-field scenarios as a generalization (RFT 9.1 perhaps, if pure scalaron were falsified​file-pbs5tcrmsvz7ndprsed51h).

These questions define a research program going forward. They involve deepening the theoretical underpinnings (unifying the scalaron with gravity at the action level, possibly involving torsion or emergent ideas) and using simulations and analyses to sharpen the predictions (especially around the quantum-classical boundary and end-state collapse). As RFT cosmology progresses, answering these will tell us if the adaptive scalaron is merely an effective proxy or a fundamental component of the universe’s fabric.

**Conclusion**

In this RFT 9.0 report, we have assembled a comprehensive theoretical picture of the **adaptive scalaron** – a single scalar field hypothesis that spans the gamut of cosmic phenomena from quantum wave-like dark matter to classical halos to modifications of gravity in the strong-field limit. We defined the scalaron’s role as an environmentally-responsive field, motivated by ultralight axion physics and scalar-tensor gravity, which inherently unifies what would otherwise be separate new physics (dark matter particles *and* modified gravity paradigms). Through detailed examination of its three behavioral regimes, we saw that the scalaron can naturally account for structure formation and galactic dynamics: forming quantum-supported solitonic cores in low-density systems, behaving as pressureless dark matter in large halos, and eventually collapsing to black holes under extreme conditions – all while being one and the same entity throughout cosmic history.

This framework offers elegant resolutions to longstanding issues (cusp-core, missing satellites) by attributing them to quantum properties of dark matter, and it does so without sacrificing concordance with high-$z$ structure or precision gravity tests thanks to the chameleon-like adaptation of the field. We also linked the scalaron concept to fundamental physics: the increase of entropy and arrow of time in structure formation aligns with the decoherence of the scalaron’s wavefunction, illustrating on a cosmological stage how classical reality can emerge from quantum beginnings. Moreover, by contemplating connections to twistor theory and emergent gravity, we placed the scalaron idea in a broader context that touches quantum gravity – an enticing prospect that solving dark matter could illuminate quantum-gravitational mysteries like black hole information.

Finally, we underscored that this is a **testable theory**. Upcoming astronomical observations – from the dynamics of dwarf galaxy cores and star streams in Finally, we underscore that this is a **testable theory**. Upcoming observations – from dwarf galaxy core dynamics and Milky Way stellar streams to gravitational-wave searches and black hole imaging – can probe the distinctive features predicted by the adaptive scalaron model. Detection of kiloparsec-scale dark matter cores, a dearth of small halo substructure, or signatures of scalar field collapse (e.g. high-frequency wave bursts) would lend strong support to this framework. Conversely, finding persistently cuspy halos or an abundance of intact subhalos would challenge the scalaron hypothesis. In this way, *RFT 9.0: The Adaptive Scalaron* not only provides a unifying theoretical foundation bridging quantum, classical, and geometric regimes of gravity, but also charts an empirical path forward. As simulations improve and data arrive, the coming years will determine whether a single adaptive scalar field can indeed be the missing key that ties together dark matter and modified gravity – illuminating the deep connection between the quantum waves that pervade our universe and the cosmic structures shaped by gravity.

**RFT 9.0: The Adaptive Scalaron – Unifying Quantum Waves, Dark Matter, and Gravity**

**Introduction**

Astrophysical and cosmological observations demand a form of “dark” gravitating matter, yet decades of searches have not revealed a conventional particle. At the same time, modifications to gravity on galactic scales (e.g. MOND-like phenomenology) hint that our understanding of gravity itself might be incomplete. A promising theoretical approach is that **dark matter and modified gravity phenomena could be two manifestations of a single underlying field**, whose behavior *adapts* to its environment​[scipost.org](https://scipost.org/submissions/2109.10928v1/#:~:text=suitable%20choice%20of%20the%20superfluid,studied%20physics%20of%20superfluidity). In this framework – developed in the Relativistic Field Theory (RFT) cosmology program – a **scalar field** dubbed the **“adaptive scalaron”** serves as this unifying agent. This scalaron field can manifest as a wave-like quantum condensate on cosmic scales, mimic cold dark matter in galaxies, and modify geometric gravity in the strong-field regime, all depending on environmental conditions.

Recent advances in scalar field cosmology support this vision. **Ultralight bosonic fields** (with masses $m \sim 10^{-22}$–$10^{-20}$ eV) have emerged as viable dark matter candidates that form macroscopic quantum states on kiloparsec scales​[nature.com](https://www.nature.com/articles/nphys2996#:~:text=galaxies%2C%20with%20only%20one%20free,%E2%89%B2%2013%20in%20our%20simulations). Such fields can exhibit wave interference and solitonic cores inside galaxies​[nature.com](https://www.nature.com/articles/nphys2996#:~:text=galaxies%2C%20with%20only%20one%20free,%E2%89%B2%2013%20in%20our%20simulations), addressing small-scale structure puzzles (like the cusp–core problem) that plague standard cold dark matter​[nature.com](https://www.nature.com/articles/nphys2996#:~:text=The%20conventional%20cold,achieve%20cosmological%20simulations%20of%20this). In parallel, scalar-tensor modifications of gravity (e.g. $f(R)$ gravity and chameleon fields) show that a scalar field coupled to curvature can reproduce galaxy dynamics without invoking heavy halos​[aanda.org](https://www.aanda.org/articles/aa/pdf/2014/01/aa21061-13.pdf#:~:text=Aims,its%20context%20and%20applied%20it)​[aanda.org](https://www.aanda.org/articles/aa/pdf/2014/01/aa21061-13.pdf#:~:text=Results,of%20galaxies%2C%20show%20evident%20correlations). The adaptive scalaron hypothesis merges these lines of thought: **a single scalar field whose effective mass and dynamics change with local density**, yielding quantum wave behavior in low-density voids and behaving like a classical gravitating mass in high-density regions. **Dark matter and modified gravity become simply different phases of one field**​[scipost.org](https://scipost.org/submissions/2109.10928v1/#:~:text=suitable%20choice%20of%20the%20superfluid,studied%20physics%20of%20superfluidity).

This document, designated *RFT 9.0: The Adaptive Scalaron*, consolidates theoretical developments and recent findings on this unifying framework. We present a formal theoretical model of the adaptive scalaron field and explore how it interpolates between quantum, classical, and geometric (curvature-coupled) regimes of gravity. We define the scalaron and its role in RFT cosmology (§2); examine its behavior in three key gravitational regimes – **fuzzy quantum wave**, **decoherent classical**, and **gravitational collapse** (§3); analyze its connection to entropy production and the cosmological arrow of time (§4); and discuss how it bridges multiple domains of theory, from dark matter phenomenology to modified gravity and twistor-geometric formulations (§5). In §6 we propose observational signatures that could test this model, including solitonic core oscillations, tidal disruption effects, collapse-generated wave signals, and black hole “hair.” Finally, in §7 we outline open theoretical questions and future directions (e.g. scalaron–curvature couplings, torsion models, collapse dynamics, and phase transitions). Throughout, we adopt an academic tone and unified notation, treating the scalaron as a field $\phi$ with a potential $V(\phi)$ and coupling functions chosen to realize the adaptive behavior. All equations are given in units where $c=\hbar=1$ (unless otherwise noted). Our goal is to provide a self-contained theoretical reference for this adaptive scalaron paradigm, which aspires to **unify dark matter and modified gravity within a single relativistic field theory**.

**The Adaptive Scalaron in RFT Cosmology**

**Definition and Origin:** *Adaptive scalaron* refers to a cosmological scalar field $\phi(x)$ whose properties (effective mass, coupling to matter/curvature, quantum coherence length) depend on the local gravitational environment. In low-density cosmic voids, $\phi$ remains light and coherent, while in high-density regions (galactic cores, stellar systems) it becomes massive or couples strongly to gravity, thus “hiding” its wave nature. The term “scalaron” is borrowed from $f(R)$ gravity, where a new scalar degree of freedom emerges from extending the Ricci scalar $R$ in the action​[inspirehep.net](https://inspirehep.net/files/2e446914ecf6a036ff82d5de6c35a1dd#:~:text=that%20power,%282015%29%20used). In Starobinsky’s $R^2$ inflation, for example, the **scalaron** is the scalar field equivalent of the $R^2$ term driving inflation​[inspirehep.net](https://inspirehep.net/files/2e446914ecf6a036ff82d5de6c35a1dd#:~:text=that%20power,%282015%29%20used). Here we generalize the notion: the adaptive scalaron is a single field postulated to pervade the cosmos, playing multiple roles – as dark matter in some contexts and as a modifier of gravity in others.

**RFT Cosmology Framework:** This idea is developed in the Relativistic Field Theory (RFT) cosmology framework​file-pbs5tcrmsvz7ndprsed51h. RFT cosmology posits that what we call “dark matter” and even “dark energy” may arise from one (or a few) fundamental fields within an extended gravitational action. In earlier RFT studies, an ultralight scalar was identified as a compelling dark matter candidate​file-c422leaz9pf3pftzffrfgv. In particular, a **“fuzzy” scalaron identified with an ultralight axion-like particle** was hypothesized​file-c422leaz9pf3pftzffrfgv. Such a field could naturally arise from high-energy theory: for instance, as a **pseudo-Nambu–Goldstone boson (axion)** from a broken global symmetry in the early universe​file-c422leaz9pf3pftzffrfgv​file-c422leaz9pf3pftzffrfgv. These axion-like scalarons have extremely small masses ($m \sim 10^{-22}$ eV) generated by non-perturbative effects, and are stable due to an approximate shift symmetry​file-c422leaz9pf3pftzffrfgv. They carry a **calculable relic abundance** set by the misalignment mechanism in the early universe​file-c422leaz9pf3pftzffrfgv, making them excellent dark matter candidates. In fact, string theory predicts a whole “axiverse” of ultralight axions; it is plausible one of the lightest has $m \sim 10^{-22}$ eV, right in the range for fuzzy dark matter​file-c422leaz9pf3pftzffrfgv. RFT cosmology 8.x studies pinpointed this **ultralight axion scalaron** as a unification candidate, with a Lagrangian of the form:

S=∫d4x−g[116πGR+12(∂μϕ)2−V(ϕ)]+Sint[ϕ,gμν,Ψi],S = \int d^4x \sqrt{-g} \left[ \frac{1}{16\pi G}R + \frac{1}{2}(\partial\_\mu \phi)^2 - V(\phi) \right] + S\_\text{int}[ \phi, g\_{\mu\nu}, \Psi\_i ],S=∫d4x−g​[16πG1​R+21​(∂μ​ϕ)2−V(ϕ)]+Sint​[ϕ,gμν​,Ψi​],

where $V(\phi)$ is a shallow potential (e.g. $m^2 \phi^2/2$ or a periodic axion potential), and $S\_\text{int}$ encodes possible non-minimal interactions of $\phi$ with curvature $R$ or matter fields $\Psi\_i$ to enable environment-dependent effects. The hallmark of the scalaron is that its **effective mass $m\_\text{eff}(\phi,\rho)$ and coupling strength vary with ambient mass density $\rho$ or curvature**. In a low-density region, $m\_\text{eff}$ is tiny, so $\phi$ coherently oscillates over large scales and mediates long-range forces; in a high-density region, $m\_\text{eff}$ grows (a “chameleon” mechanism​[aanda.org](https://www.aanda.org/articles/aa/pdf/2014/01/aa21061-13.pdf#:~:text=Aims,its%20context%20and%20applied%20it)), so $\phi$ is short-range and effectively pressureless.

**Unifying Dark Matter and Modified Gravity:** In this single-field picture, the scalaron underlies both the dark matter phenomena in cosmic structure and any apparent modifications to gravity in regimes where dark matter is scarce. This is conceptually illustrated by analogies to *phase transitions*: **the scalaron has multiple phases**. In one phase (low-density), it behaves like a quantum condensate with wave-like (or superfluid) properties; in another phase (intermediate-density), it behaves like a classical collisionless dust; in yet another (extreme density/curvature), it becomes strongly coupled to geometry, altering the effective gravitational law. Notably, **Khoury’s theory of superfluid dark matter** embodies a similar paradigm: galactic halos of an axion-like particle condense into a superfluid core whose phonon excitations give rise to MOND-like modified gravity, while outside the core the particles are normal cold dark matter​[scipost.org](https://scipost.org/submissions/2109.10928v1/#:~:text=In%20these%20lectures%20I%20describe,studied%20physics%20of%20superfluidity). *“Thus the dark matter and modified gravity phenomena represent different phases of a single underlying substance, unified through the rich physics of superfluidity.”*​[scipost.org](https://scipost.org/submissions/2109.10928v1/#:~:text=suitable%20choice%20of%20the%20superfluid,studied%20physics%20of%20superfluidity) Although the scalaron in RFT 9.0 is an ultralight ($\sim10^{-22}$ eV) field (distinct from the $m\sim 1$ eV scale of superfluid DM models​[scipost.org](https://scipost.org/submissions/2109.10928v1/#:~:text=In%20these%20lectures%20I%20describe,different%20phases%20of%20a%20single)), the principle is analogous: **one field, multiple behaviors**. The scalaron can form condensate cores that produce effective long-range forces on baryons (mimicking modified gravity), while most of its mass still behaves as diffuse dark matter on larger scales. Environmental dependence is key: the field’s behavior is *adaptive* – a dense concentration of baryonic or dark mass can trigger the scalaron to transition from one regime to another (for example, suppressing its gradient pressure and acting like a classic gravitational potential). In modified gravity terms, the scalaron acts as an extra degree of freedom that is active on galactic scales but screened in the solar-neighborhood, much like chameleon fields that **vary with local matter density**​[aanda.org](https://www.aanda.org/articles/aa/pdf/2014/01/aa21061-13.pdf#:~:text=Aims,its%20context%20and%20applied%20it).

To summarize, the adaptive scalaron is a cosmological scalar field motivated by high-energy theory (e.g. an axion from the string axiverse)​file-c422leaz9pf3pftzffrfgv that **constitutes the dark matter** while simultaneously providing a mechanism for **environment-dependent modifications of gravity**. In the next sections, we detail how this single scalar field behaves across different gravitational regimes and how it provides a continuous interpolation between quantum, classical, and relativistic-gravity domains.

**Regimes of Scalaron Dynamics Across Scales**

A central thesis of the adaptive scalaron framework is that a single field $\phi$ can reproduce the appropriate behavior in *three distinct gravitational regimes*: (i) the **fuzzy quantum wave regime** prevalent in low-density environments (or early linear universe), (ii) the **decoherent classical regime** in virialized galactic and cluster halos, and (iii) the **collapse/black-hole regime** in extreme overdensities. We now characterize the scalaron’s behavior in each regime, highlighting how the field’s equations of motion reduce to the expected physics.

**Fuzzy Wave Regime (Quantum Coherent Phase in Voids)**

In the most diffuse environments – cosmic voids or the outskirts of halos where the ambient density of matter is extremely low – the scalaron remains in a **quantum wave-dominated regime**. Here $\phi$ can be thought of as a *giant coherent wavefunction* oscillating in its potential. The de Broglie wavelength $\lambda\_{\rm dB}$ of the field (inversely proportional to momentum) is huge, often kiloparsecs, exceeding the size of small dark matter structures. As a result, the scalaron does not fragment into particles but forms a **Bose–Einstein condensate (BEC) on cosmological scales**​[nature.com](https://www.nature.com/articles/nphys2996#:~:text=The%20conventional%20cold,achieve%20cosmological%20simulations%20of%20this). The uncertainty principle (quantum pressure) counters gravity below a Jeans scale, preventing arbitrarily small clumps​[nature.com](https://www.nature.com/articles/nphys2996#:~:text=The%20conventional%20cold,achieve%20cosmological%20simulations%20of%20this). This regime corresponds to the idea of **“fuzzy dark matter”** or **wave dark matter (ψDM)** in the literature​[nature.com](https://www.nature.com/articles/nphys2996#:~:text=The%20conventional%20cold,achieve%20cosmological%20simulations%20of%20this).

**Wave Mechanics:** In this phase, the dynamics of $\phi$ are well-described by the coupled Schrödinger–Poisson (SP) or Gross-Pitaevskii equations. The scalaron’s gravitational potential $\Phi$ and wavefunction $\psi \sim e^{-i m t}\phi$ obey:

iℏ∂ψ∂t=−ℏ22m∇2ψ+mΦψ,∇2Φ=4πG(ρb+m∣ψ∣2),i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + m \Phi \psi, \qquad \nabla^2 \Phi = 4\pi G (\rho\_b + m|\psi|^2),iℏ∂t∂ψ​=−2mℏ2​∇2ψ+mΦψ,∇2Φ=4πG(ρb​+m∣ψ∣2),

where $|\psi|^2$ gives the scalaron density and $\rho\_b$ is any ordinary (baryonic) density present. In voids, $\rho\_b \approx 0$. The key feature is that $\psi$ remains *single-valued with a well-defined global phase*, i.e. a coherent state. Large-scale structure formation proceeds similarly to cold dark matter on large scales, but on small scales the wave nature becomes evident. Indeed, cosmological simulations of wave dark matter show that on **large scales the structure is indistinguishable from CDM**, but **inside halos the behavior differs markedly**​[nature.com](https://www.nature.com/articles/nphys2996#:~:text=galaxies%2C%20with%20only%20one%20free,%E2%89%B2%2013%20in%20our%20simulations). Quantum interference effects dominate the internal structure of halos in this regime.

**Solitonic Cores:** A hallmark of this fuzzy regime is the formation of **solitonic cores** – self-supported, stationary wave solutions that sit in the centers of gravitational potential wells. As a halo grows by accretion or mergers, the central region of the scalaron field can relax into the lowest-energy eigenstate (a soliton). **Simulations demonstrate that quantum interference of the coherent field naturally produces a solitonic core surrounded by a halo of wave interference “granules.”** In a representative result by Schive *et al.*, *“quantum interference forms solitonic cores surrounded by extended halos of fluctuating density granules.”*​[nature.com](https://www.nature.com/articles/nphys2996#:~:text=galaxies%2C%20with%20only%20one%20free,%E2%89%B2%2013%20in%20our%20simulations) The soliton is a Bose–Einstein condensate core where quantum pressure (from the $\nabla^2\psi$ term) balances gravity. Its density profile is well-fit by $\rho\_{\rm soliton}(r) \propto [1 + 0.091 (r/r\_c)^2]^{-8}$, with a core radius $r\_c$ scaling inversely with the core mass and the particle mass. Surrounding the soliton, the remainder of the halo consists of a chaotic interference pattern of excited states – these manifest as time-dependent, spatially granular density fluctuations often described as a “quantum fog.” **The entire halo can thus be viewed as a standing wave (core) plus a superposition of traveling waves (envelope).** This picture is vividly supported by simulations: for instance, a fuzzy dark matter halo shows a dense solitonic center and an envelope akin to an NFW profile but with continual small-scale fluctuations​[osti.gov](https://www.osti.gov/biblio/1851580#:~:text=We%20report%20that%20a%20fuzzy,As%20the).

**Examples:** In a dwarf galaxy-size halo (virial mass $\sim10^9 M\_\odot$), an ultralight scalaron ($m \sim 10^{-22}$ eV) would form a core of radius a few hundred parsecs with a density $\sim10^2$–$10^3 M\_\odot/\text{pc}^3$, much shallower than a CDM cusp, addressing the core–cusp problem​[nature.com](https://www.nature.com/articles/nphys2996#:~:text=The%20conventional%20cold,achieve%20cosmological%20simulations%20of%20this). In cosmic voids, where large-scale potential wells are absent, the scalaron field may remain almost uniform, with only gentle acoustic oscillations. One can imagine a void as filled with a very low-amplitude, nearly homogeneous $\phi$ field oscillating at its natural frequency $m$; effectively, a patch of vacuum with a cold axion background. In such regions the field’s quantum coherence is maximal (phase correlations over Mpc scales). There could even exist **interference patterns on intergalactic scales**, e.g. filamentary standing-wave nodes, though these would be subtle. Overall, the fuzzy wave regime ensures that on the largest scales the scalaron behaves like a smooth fluid (preserving the successes of $\Lambda$CDM on CMB and galaxy clustering), while on small scales it introduces quantum pressure that prevents over-densification of dark matter and naturally forms cores​[nature.com](https://www.nature.com/articles/nphys2996#:~:text=The%20conventional%20cold,achieve%20cosmological%20simulations%20of%20this).

**Decoherent Classical Regime (Galaxies and Clusters)**

As one considers environments of higher density – for example, the interior of large dark matter halos such as the Milky Way or clusters of galaxies – the scalaron’s behavior transitions toward that of **classical, pressureless dark matter**. This is the regime of a **decohered scalaron**, where the field’s phase coherence is largely lost due to complex dynamics and interactions. Gravitational collapse and virialization excite many modes of the field, populating a quasi-thermal ensemble of wave states rather than a single coherent ground state. In plainer terms, **the scalaron field in a galaxy behaves as a collection of particles** for most purposes, even though fundamentally it is a wave.

**Core–Halo Structure:** Importantly, the transition is not total – the innermost region of halos can remain in the quantum solitonic state, but the outer regions become effectively classical. High-resolution studies show a **distinct “core–halo” bifurcation** in fuzzy dark matter halos: *“a fuzzy dark matter halo consists of a soliton core close to the center and a Navarro-Frenk-White–like density profile in the outer region.”*​[osti.gov](https://www.osti.gov/biblio/1851580#:~:text=We%20report%20that%20a%20fuzzy,As%20the) The inner soliton (ground state) is the vestige of quantum coherence, while the outer halo is comprised of excited states that have decohered into a virialized cloud. The outer density profile approximates the NFW form seen in CDM, meaning the scalaron reproduces the successful empirical profile of large halos beyond the core radius​[osti.gov](https://www.osti.gov/biblio/1851580#:~:text=We%20report%20that%20a%20fuzzy,As%20the). In effect, **outside the core the scalaron’s quantum pressure is negligible** and the field behaves as if it were pressureless cold matter. This is why large-scale and high-density structures (like clusters) are not disrupted by quantum effects – their de Broglie wavelength is tiny compared to the system, so one recovers the classical limit.

**Decoherence Mechanism:** The loss of coherence can be understood through wave interference and coupling to environment. When many waves overlap with uncorrelated phases (e.g. during hierarchical merging of subhalos, or interaction with baryonic potential variations), the relative phase information becomes scrambled. One can treat different patches of the scalaron field as having independently evolving phases once separated by nonlinear dynamics (much like how quantum phases decohere when a system becomes entangled with a chaotic environment). Quantum decoherence is effectively an entropy-increasing process – the initially “pure” state of the field (low entropy, single phase relation) becomes a “mixed” state with higher entropy (randomized phases)​[en.wikipedia.org](https://en.wikipedia.org/wiki/Arrow_of_time#:~:text=match%20at%20L436%20decoherence%20is,In%20the%20language). Practically, this means the interference fringes in the halo act like pseudoparticles. They do not maintain a stable phase relation and instead behave like a **cloud of collisionless DM pseudoparticles**.

For a given halo, one useful picture is the **eigenmode decomposition**: the halo’s scalar field can be expanded in a set of eigenfunctions of the gravitational potential. The lowest mode is the soliton; higher modes correspond to excited, higher-energy states. Due to gravitational interactions, these modes are continually exchanging energy (through interference and tidal perturbations), so any initial phase alignment dissipates. Li, Hui & Yavetz (2021) showed that the phenomena of **soliton core oscillations and random walks** can be explained by interference of the ground state with a spectrum of excited states​[osti.gov](https://www.osti.gov/biblio/1851580#:~:text=Previous%20investigations%20found%20that%20the,of%20the%20excited%20states%2C%20the). As long as significant excited state amplitude is present, the core will oscillate and move around, indicating the system is not in a single coherent eigenstate but a superposition​[osti.gov](https://www.osti.gov/biblio/1851580#:~:text=of%20numerical%20simulations%2C%20we%20show,As%20the). Over time, one expects some phase averaging; indeed, if a subhalo is stripped of its outer layers (reducing excited state amplitude), the soliton core’s oscillations diminish, approaching a more steady state​[osti.gov](https://www.osti.gov/biblio/1851580#:~:text=,deducing%20constraints%20from%20stellar%20heating). This is essentially **decoherence in action**: removing the “environment” (outer halo waves) leaves the core closer to a pure state.

**Effective Dust Behavior:** In the bulk of a galaxy or cluster halo, the scalaron can be treated as a classical **pressureless fluid** on macroscopic scales. Cosmologically, it means structure formation with scalaron dark matter will follow the same evolution as $\Lambda$CDM at late times – a key requirement for any dark matter model. The halo mass function, large-scale clustering, and dynamics of clusters (massive enough that their core radius is much smaller than their virial radius) should remain as in CDM. Observers would infer “cold dark matter” behavior in lensing and dynamics of clusters. In galaxies, the difference appears mainly in the inner kpc or so (where a core replaces a cusp). At larger radii, fits to rotation curves under scalaron dark matter are similar to CDM (aside from slightly lower concentration for low-mass halos due to the suppression of small-scale power)​[nature.com](https://www.nature.com/articles/nphys2996#:~:text=The%20conventional%20cold,achieve%20cosmological%20simulations%20of%20this). The scalaron thus passes the basic phenomenological tests: it forms structure on large scales nearly identical to CDM​[nature.com](https://www.nature.com/articles/nphys2996#:~:text=galaxies%2C%20with%20only%20one%20free,%E2%89%B2%2013%20in%20our%20simulations), and on small scales it naturally fixes some CDM problems by virtue of its quantum-derived core and the dearth of very low-mass halos (discussed further in §5.1).

It is worth noting that as the field enters this classical regime, the notion of a single field still holds – we are not switching to a different substance, just a different behavior of $\phi$. One could in principle recover the classical evolution by taking the Wigner transform of the wavefunction to define a pseudo phase-space density $f(\mathbf{x}, \mathbf{v})$. In the classical regime, $f$ approximately obeys the collisionless Boltzmann equation (or Vlasov–Poisson), just as CDM N-body particles would. Therefore, simulations of scalaron dark matter on cluster scales show **no significant deviation from CDM predictions** for large-scale structure​[nature.com](https://www.nature.com/articles/nphys2996#:~:text=galaxies%2C%20with%20only%20one%20free,%E2%89%B2%2013%20in%20our%20simulations). Only by looking at fine-grained structure (e.g. the granularity of the halo or the oscillation of the core) would one notice that the underlying dark matter is a wavy scalar field and not literal point particles.

In summary, the **decoherent regime** is where the adaptive scalaron acts as standard dark matter. It **coexists** with the quantum regime in every halo (quantum core + classical envelope) and ensures continuity with well-tested gravitational physics in systems like galaxy clusters, while still providing distinctive differences at smaller scales (galactic cores, satellites) that can test the model.

**Collapse and Black Hole-Bound Regime (Strong-Gravity Limit)**

The third regime occurs in regions of **extreme density or gravitational potential**, where even the classical dark matter behavior transitions – namely, when the scalaron forms compact objects approaching the Schwarzschild radius scale. This can happen, for instance, if a scalaron solitonic core grows beyond a certain critical mass and undergoes gravitational collapse. In the universe, such situations might arise in the centers of large galaxies or in collapsing primordial overdensities. The end result is the formation of a **black hole (BH)**, but with unique features stemming from the scalaron’s field nature. We refer to this as the **collapse/BH-bound regime**, characterized by **soliton instability, scalar radiation, and transient “hair” before settling into a black hole**.

**Soliton Instability and Collapse:** A self-gravitating boson (scalar) configuration is stable only up to a maximum mass. For a free (non-self-interacting) scalaron star (often called a boson star or axion star), there is a well-known **Kaup limit**: $M\_{\rm max} \approx 0.633, M\_\text{Pl}^2/m$ (where $M\_\text{Pl}$ is the Planck mass and $m$ the particle mass)​[link.springer.com](https://link.springer.com/article/10.1140/epjc/s10052-019-6940-z#:~:text=,interaction%20with%20potential%20%5C%28V%28%5Cphi). For $m\sim 10^{-22}$ eV, this gives an enormous $M\_{\rm max} \sim 10^{12} M\_\odot$ – interestingly, around the scale of a large galaxy’s dark matter halo. However, in practice, environmental factors and self-interactions alter this number. If the scalaron has even tiny self-interactions (as an axion would), an **“axion star”** can become unstable at much lower masses. Attractive self-interactions (like the cosine axion potential) tend to decrease the stability limit, causing collapse at lower masses via a bosenova-type instability​[link.springer.com](https://link.springer.com/article/10.1140/epjc/s10052-019-6940-z#:~:text=formation,in%20the%20case%20of%20dense). Repulsive self-interactions (e.g. a $\lambda \phi^4$ term with $\lambda>0$) increase the max mass considerably​[link.springer.com](https://link.springer.com/article/10.1140/epjc/s10052-019-6940-z#:~:text=concluded%20that%20in%20the%20free,28%5D%29.%20On%20the), potentially preventing collapse except in extremely massive configurations.

We posit that for the cosmologically relevant scalaron, the parameters are such that **halo cores in most galaxies are stable (sub-critical), but in extreme cases (perhaps at centers of massive galaxies or during mergers) a core can accumulate above the critical threshold**. When that threshold is crossed, the solitonic core can no longer support itself against gravity. What follows is a complex **gravitational collapse**. Numerical studies of axion star collapse show a two-stage process: first a mild collapse that triggers partial expulsion of mass (an “axion nova”), then – if enough mass remains – a final collapse to a BH​[link.springer.com](https://link.springer.com/article/10.1140/epjc/s10052-019-6940-z#:~:text=mass%2C%20leading%20to%20an%20instability,6). In the first stage, as the core compresses, its central density rises rapidly and the field oscillations become relativistic. The scalar field can begin radiating away kinetic energy in the form of scalar waves (analogous to how a collapsing BEC can eject particles). This **scalar radiation** carries away some of the mass and prevents an immediate total collapse. However, if the mass is well above critical, the expulsion is not enough to halt collapse. Eventually, a black hole forms at the center.

**Black Hole Formation and Transient Scalar Hair:** When the black hole forms, most of the scalaron’s mass within the event horizon simply becomes part of the black hole (increasing its mass). In classical GR, a stationary black hole cannot hold onto a free scalar field – the no-hair theorem dictates that a minimally-coupled scalar either falls in or radiates away. However, **during and shortly after the collapse, one can have “transient scalar hair.”** This refers to scalar field configurations outside the horizon that are non-zero for some time. For example, some of the scalar field that did not cross the horizon immediately might linger as a cloud or as outgoing waves. Because the system was highly coherent (a soliton) just prior to collapse, it is plausible that a portion of $\phi$ ends up in quasi-bound states around the nascent black hole (similar to quasi-normal modes). Over time, this hair will dissipate: massive scalar field modes will radiate their energy either into the black hole (settling inside the horizon) or to infinity as scalar (or gravitational) radiation. But the **decay can be slow** if the field is massive and gravitationally bound – in effect, the black hole + scalar system might form a gravitational atom (like the proposed axion clouds around spinning black holes via superradiance). In our case, the black hole is formed *from* the scalar, so initially there is a lot of scalar around. This scenario is ripe for rich dynamics: the scalar cloud can undergo oscillations, tidal disruption by the hole’s gravity, and resonant excitations.

An intriguing possibility is if the scalaron has self-interactions, the collapsing object might not directly produce a quiet black hole but instead an **explosive event**. There are analogies to condensed matter: a collapsing axion Bose star might undergo bosenova bursts observed as intense radiation (in analogy to laboratory BEC collapse due to attractive interactions). For cosmological scalaron, this could translate to a **burst of scalar radiation and high-frequency gravitational waves** at the moment of collapse.

**Gravitational Wave Emission:** The collapse of a scalaron core into a BH is a highly dynamic process, and like any asymmetric collapse, it should emit **gravitational waves (GWs)**. However, the frequency of these waves is set by the timescale of the collapse – which for such light scalars can be extremely high frequency (kHz to GHz). For instance, one analysis suggests that axion clumps can produce “fast gravitational wave bursts” with frequencies on the order of $\frac{1}{2}m/\pi$ (in physical units)​[link.aps.org](https://link.aps.org/doi/10.1103/PhysRevD.104.103009#:~:text=%28FRBs%29,more%20evidence%20for%20the%20axion). For $m=10^{-22}$ eV, this is $\sim 10^{-7}$ Hz – actually very low (months period) if taken literally. But if smaller, denser clumps ($m$ effectively larger or collapse happening on sub-second times) are considered, frequencies in kHz range could occur​[link.aps.org](https://link.aps.org/doi/10.1103/PhysRevD.104.103009#:~:text=%28FRBs%29,more%20evidence%20for%20the%20axion). In any case, these signals are likely difficult to detect with current detectors, but they are conceptually important: **a collapsing scalaron could produce a unique gravitational wave signature** distinct from, say, the merger of two black holes. It might be a burst or an “echo” following a primary collapse.

**Endpoint:** Ultimately, the end state of the collapse regime is a **black hole (with mass mostly from the scalaron) plus possibly a residual halo of scalaron material**. The black hole will obey the usual laws (e.g. if it’s non-rotating, it is a Schwarzschild BH characterized just by its mass and possibly charge). The scalaron field far away will revert to whatever the ambient conditions dictate (e.g. the halo outside the new BH might still have a solitonic structure if some scalaron remains bound outside). Over time, any scalar “hair” decays – calculations show that massive scalar perturbations around a BH have quasi-normal modes that damp out. However, one exciting aspect is that if the black hole is spinning, it can *regenerate* a scalar cloud via superradiant instabilities – essentially acting like a particle collider for the scalaron. That strays beyond our current scope, but it means **black holes could be laboratories for the scalaron**, leading to phenomena like gravitational atom spectra or persistent oscillating fields (with potential astrophysical signals).

**Transient vs Permanent Modified Gravity:** In this strong-field regime, does the scalaron produce any **modified gravity effect**? Potentially yes, but only transiently. During collapse, the scalar field’s stress-energy is complex and could cause departures from pure vacuum GR in the surrounding spacetime (a nontrivial energy-momentum distribution outside the horizon). This could be considered a **modified gravity regime** in the sense that the solution isn’t just Schwarzschild; there is a scalar field contributing to the metric. However, as the scalar dissipates or falls in, one returns to standard GR with a Schwarzschild (or Kerr) metric for the BH. So the modifications to gravity are not long-lived unless some mechanism stabilizes the scalar field outside (which typically would require either a special coupling or a non-decay into the BH; some models of scalar–tensor gravity do admit *hairy black holes*, but those often involve additional potentials or breaking of no-hair conditions).

In summary, the collapse regime shows the scalaron’s ability to behave like a **relativistic field** in strong gravity: it can form black holes just as normal matter can, but with accompanying phenomena of scalar wave emission and temporary hair. This regime underscores that the adaptive scalaron truly spans from **quantum to classical to relativistic gravity**: it is born as a quantum wave, lives as a classical halo, and dies (in a sense) as a contribution to a black hole. The formation of black holes also ties into entropy and information in ways we discuss next.

**Entropy, Decoherence, and the Arrow of Time**

An appealing aspect of the adaptive scalaron framework is that it offers a concrete picture of how *quantum cosmological initial conditions* evolve into *classical structure* with an associated increase in entropy – illuminating the **cosmological arrow of time**. Here we analyze the flow of entropy and role of decoherence as the scalaron progresses through its different regimes.

**Low-Entropy Beginnings:** At early times, or in large voids, the scalaron field can be approximated as a homogeneous, coherent state – essentially a zero-entropy configuration. In quantum terms, if the field is in a pure state (e.g. the BEC ground state), its entropy (von Neumann entropy of the density matrix) is minimal. Cosmologically, this aligns with the idea that the early Universe (nearly homogeneous density, including dark matter) had remarkably low entropy in its gravitational degrees of freedom​[preposterousuniverse.com](https://www.preposterousuniverse.com/blog/2007/12/03/arrow-of-time-faq/#:~:text=The%20observed%20macroscopic%20irreversibility%20is,the%20origin%20of%20the%20universe)​[preposterousuniverse.com](https://www.preposterousuniverse.com/blog/2007/12/03/arrow-of-time-faq/#:~:text=our%20current%20cosmological%20horizon%2C%20that%E2%80%99s,all%20of%20the%20matter%20in). All the scalaron quanta oscillating in phase is analogous to a system at zero temperature. This is consistent with the **Past Hypothesis** in cosmology: the Universe started in a special low-entropy state (smooth, no structure). Indeed, if the scalaron is an axion produced by misalignment, its initial condition is a spatially constant field value with some zero-velocity – a very ordered state.

**Structure Formation and Entropy Increase:** As time progresses, small perturbations in the scalaron grow under gravity (just like perturbations in CDM would). Overdense regions become halos, and within halos the field decoheres into multiple modes. **Every merger, virialization event, or interaction effectively increases the coarse-grained entropy of the scalaron field.** The formerly coherent phases become scrambled, corresponding to a move from a pure state to a mixed state (when only gross degrees of freedom are observed)​[en.wikipedia.org](https://en.wikipedia.org/wiki/Arrow_of_time#:~:text=match%20at%20L436%20decoherence%20is,In%20the%20language). In thermodynamic language, the system thermalizes gravitationally – though not in temperature, but in dynamical equilibrium. The **thermodynamic arrow of time** is manifest: entropy (disorder in phase space) increases as structures form. This is tightly connected with **decoherence**: quantum decoherence is essentially an entropy increase at the microscopic level​[en.wikipedia.org](https://en.wikipedia.org/wiki/Arrow_of_time#:~:text=match%20at%20L436%20decoherence%20is,In%20the%20language). When the scalaron’s wavefunction entangles with complicated environmental degrees (like multi-stream motions, baryonic clumps, etc.), the relative phases become effectively random. *“Decoherence is a form of increase in microscopic disorder – in short, decoherence increases entropy.”*​[en.wikipedia.org](https://en.wikipedia.org/wiki/Arrow_of_time#:~:text=match%20at%20L436%20decoherence%20is,In%20the%20language). Thus the growth of cosmic structures provides a concrete mechanism for decoherence: the many-body gravitational interaction plays the role of an environment that irreversibly correlates and scrambles the phases of the field. The process is unidirectional in time – once phases are lost, it’s practically impossible to spontaneously recohere the field into its original pure state (that would require an extraordinarily tuned time-reversal of all interactions, prohibited by the second law)​[en.wikipedia.org](https://en.wikipedia.org/wiki/Arrow_of_time#:~:text=decoherence%20is%20a%20form%20of,In%20the%20language).

**Arrow of Time:** The cosmological arrow of time (the direction in which the universe becomes more structured and more entropic) is intimately tied to gravity and thus to the behavior of dark matter. In our scenario, the scalaron starts in a low-entropy state and ends up contributing to extremely high entropy structures (black holes). Roger Penrose famously argued that gravity’s tendency to clump matter leads to the arrow of time, since a homogeneous gas has low gravitational entropy while a collection of black holes has maximal entropy​[preposterousuniverse.com](https://www.preposterousuniverse.com/blog/2007/12/03/arrow-of-time-faq/#:~:text=our%20current%20cosmological%20horizon%2C%20that%E2%80%99s,all%20of%20the%20matter%20in). Our adaptive scalaron provides a step-by-step realization: initial smooth scalar field (low entropy) -> halo with core and incoherent halo (higher entropy) -> possibly black hole formation (very high entropy). For example, consider the entropy in units of Boltzmann’s constant: A single 10^9 $M\_\odot$ soliton core has some entropy associated with its excitations, but if that core collapses into a $10^9 M\_\odot$ black hole, the entropy jumps enormously (a Schwarzschild BH of that mass has $S \sim 10^{90}$ in dimensionless units)​[preposterousuniverse.com](https://www.preposterousuniverse.com/blog/2007/12/03/arrow-of-time-faq/#:~:text=our%20current%20cosmological%20horizon%2C%20that%E2%80%99s,all%20of%20the%20matter%20in). This dwarfs any entropy previously in the dark matter distribution. The **entropy of the universe increases by orders of magnitude as scalaron cores collapse into black holes** – an irreversible step cementing the arrow of time.

**Entropy Flow in Phases:** We can delineate entropy flow through the scalaron’s phases: (i) *Quantum-coherent phase:* minimal entropy (all particles in one mode). (ii) *Decoherent halo phase:* entropy produced as modes populate and phases randomize. The dark matter (scalaron) effectively gains an entropy akin to phase-space volume occupied by its granules. (This can be quantified by an effective phase-space density decrease – the wave nature sets a max phase-space density, and as modes excite, the system “warms up” in a sense.) (iii) *Black hole phase:* a huge entropy increase concentrated in the BH horizon. Notably, most of that entropy is gravitational (in the geometry), not carried by the scalar field’s degrees of freedom anymore. One can think of it as the scalaron’s entropy being dumped into the spacetime itself (the BH). This might be relevant to the information problem: the pure state of many scalar particles turns into the mixed state of a Hawking radiation/BH horizon ensemble.

**Cosmological Arrow and Quantum Arrow:** It has been argued that the **thermodynamic arrow of time (entropy increase)** gives rise to the **quantum arrow (decoherence)**​[en.wikipedia.org](https://en.wikipedia.org/wiki/Arrow_of_time#:~:text=,6%20Quantum%20arrow%20of%20time)​[en.wikipedia.org](https://en.wikipedia.org/wiki/Arrow_of_time#:~:text=The%20conventional%20approach%20is%20to,dynamics%20is%20assumed%20to%20be). In our scenario, they are indeed aligned: as structures form and entropy increases, the scalaron decoheres. The arrow of time is evident – we see structure (and decoherence) growing in one time direction (forward). Time-reversing a virialized halo full of granular interference patterns would require them all to converge and phase-align into a smooth field – a fantastically improbable “entropy-decreasing” event.

In conclusion, the adaptive scalaron model is fully compatible with and illustrative of the cosmological arrow of time. It starts in an ordered state and naturally evolves into disordered states. The *decoherence of the scalaron field links microscopic quantum irreversibility to macroscopic gravitational irreversibility*. This not only provides a satisfying narrative (the universe transitions from quantum to classical in the dark sector as time goes on), but also has practical implications: once decohered on small scales, the scalaron can be treated classically, justifying why we can use classical N-body simulations for most structures. However, tiny residual quantum effects (like core oscillations) remain as relics of the low-entropy past. The entropy considerations additionally highlight that **black holes are the final depositories of the scalaron’s information**, pushing us to consider how (or if) any information about the field’s initial quantum state might be encoded or lost – an issue we revisit when discussing “wavefunction memory” in §7.

**Bridging Theoretical Domains: From Dark Matter to Geometry**

One of the compelling strengths of the adaptive scalaron hypothesis is how it serves as a bridge between disparate theoretical domains. Here we examine these connections in detail, showing that the scalaron provides a single framework that touches upon **dark matter phenomenology**, **modified gravity**, **quantum coherence vs. classicality**, and even **twistor/geometric formulations of gravity**. We discuss each in turn.

**Dark Matter Phenomenology and Small-Scale Structure**

The scalaron was originally motivated by dark matter problems, so it must successfully reproduce (or improve upon) the phenomenology of cold dark matter. We find that it indeed captures the large-scale successes of CDM while offering solutions to several **small-scale puzzles**:

* **Large-Scale Structure:** As noted, on scales of clusters and beyond, the scalaron behaves indistinguishably from standard CDM​[nature.com](https://www.nature.com/articles/nphys2996#:~:text=galaxies%2C%20with%20only%20one%20free,%E2%89%B2%2013%20in%20our%20simulations). It has the same linear growth and clustering properties (for appropriate initial conditions). Therefore, it matches observations like the cosmic microwave background (which constrains the matter power spectrum on large scales) and galaxy two-point correlations, etc. The introduction of a quantum Jeans scale (due to the scalar’s effective “quantum pressure”) only affects the very small-scale cutoff of the power spectrum​[nature.com](https://www.nature.com/articles/nphys2996#:~:text=The%20conventional%20cold,achieve%20cosmological%20simulations%20of%20this). For $m \sim 10^{-22}$ eV, structure formation is **delayed on very small scales**: no halos below ~$10^7$–$10^8 M\_\odot$ (since perturbations below a kpc scale are wiped out by the wave support)​[nature.com](https://www.nature.com/articles/nphys2996#:~:text=The%20conventional%20cold,achieve%20cosmological%20simulations%20of%20this). This is actually desirable because CDM arguably over-predicts low-mass halos.
* **Cusp–Core Problem:** Many dwarf galaxies and low-surface-brightness galaxies exhibit shallow density cores instead of the steep cusps predicted by CDM. The scalaron’s fuzzy wave regime naturally yields **kiloparsec-scale cores** in dwarf halos​[nature.com](https://www.nature.com/articles/nphys2996#:~:text=The%20conventional%20cold,achieve%20cosmological%20simulations%20of%20this). The solitonic core density profiles match the observed cores in many cases (with core sizes scaling inversely with halo mass, qualitatively similar to trends needed to explain bigger cores in smaller galaxies). For example, a typical $10^{10} M\_\odot$ halo might get a $\sim1$ kpc core of the right density to fit dwarf spheroidal kinematics. This happens *without fine-tuning*, emerging from the physics of the scalar field.
* **Missing Satellites Problem:** CDM predicts a wealth of subhalos around Milky Way-like galaxies, far more than the number of observed dwarf satellite galaxies. The scalaron addresses this in two ways: (1) The initial power spectrum cutoff means fewer small halos form in the first place (similar to warm dark matter). (2) Even those that form can be more easily tidally destroyed (see next point), further reducing the count. Thus, the **abundance of satellite halos is naturally suppressed**. Simulations of fuzzy dark matter show that for $m \approx 10^{-22}$ eV, the halo mass function is significantly reduced below $\sim10^8 M\_\odot$, bringing the number of subhalos in line with observed satellites for a Milky Way-sized host, without conflicting with larger-scale structure​[nature.com](https://www.nature.com/articles/nphys2996#:~:text=The%20conventional%20cold,achieve%20cosmological%20simulations%20of%20this).
* **Tidal Stability and Dynamical Friction:** Because subhalos of scalaron dark matter have lower concentrations (especially if their cores are big relative to their size) and because they can undergo ram-pressure-like wave stripping, they may disrupt more easily when orbiting a larger host. The study by Du *et al.* found that a solitonic core under tidal stress will undergo a **runaway disruption once it loses enough mass**, dissolving faster than a CDM subhalo would​[arxiv.org](https://arxiv.org/abs/1801.04864#:~:text=of%20tidal%20disruption%2C%20highlighting%20the,the%20minimum%20mass%20of%20cores). This implies that observed stellar streams might show fewer perturbations (gaps) from subhalos, consistent with some current stream observations that hint at a lack of heavy perturbers. Additionally, fuzzy subhalos impart less dynamical friction (their cores fluffier), possibly alleviating issues like the “too big to fail” problem (the most massive CDM subhalos predicted seem to not host visible dwarfs – in scalaron DM those subhalos might have been erased or never formed).
* **Galaxy Formation Timing:** With small-scale power suppressed, galaxy formation, particularly for tiny protogalaxies, is delayed. Simulations indicate galaxy formation in a fuzzy DM universe starts later (no objects before $z\sim 10$–$20$ for $m\sim10^{-22}$ eV)​[nature.com](https://www.nature.com/articles/nphys2996#:~:text=allow%20us%20to%20determine%20Image,%E2%89%B2%2013%20in%20our%20simulations). This could be related to recent JWST observations of early galaxies: if JWST had found an *excess* of dwarf galaxies at high redshift, it would challenge fuzzy DM. Conversely, if it finds fewer small galaxies than CDM expects, that supports a scalaron-like DM. This is a test in progress.

In summary, the scalaron as dark matter **preserves large-scale concordance** (like CDM) but **mitigates small-scale conflicts** by virtue of its finite quantum pressure and wave dynamics. The presence of a consistent quantum core–halo structure across halos is a clear prediction: e.g., a Milky Way-sized halo should harbor a $\sim 100$ pc dense soliton (perhaps affected by the central BH)​[sciencedirect.com](https://www.sciencedirect.com/science/article/abs/pii/S2212686419303334#:~:text=Dynamical%20evidence%20of%20a%20dark,is%20smaller%2C%20pc%2C%20of%20mass), and ultra-faint dwarf galaxies should be supported by 0.5–1 kpc cores (leading to cored velocity profiles). These phenomenological successes are a strong motivator for the scalaron theory, and future observations (e.g. precision stellar kinematics in dwarfs, strong gravitational lensing for halo profiles) can further test these predictions.

**Connections to Modified Gravity (Scalar-Tensor Theories and Screening)**

While the scalaron mostly behaves as dark matter, in certain regimes it can mimic modified gravity by contributing an additional force or altering the effective gravitational constant. Here we connect the scalaron to known frameworks of modified gravity:

* **Scalar-Tensor Equivalence:** It is well known that many modified gravity theories can be cast as General Relativity plus a scalar field. For example, **$f(R)$ gravity** (where the Lagrangian is a function of Ricci scalar) can be rewritten as GR with an extra scalaron field $\phi = f'(R)$ that mediates a force​[inspirehep.net](https://inspirehep.net/files/2e446914ecf6a036ff82d5de6c35a1dd#:~:text=that%20power,%282015%29%20used). In our case, we introduced the scalaron as a physical field from the outset, but one could imagine that what we call $\phi$ might partly originate from such an $f(R)$ term or similar. Indeed, the *original scalaron* (Starobinsky’s) was massive ($10^{-12}$ GeV) and drove inflation​[inspirehep.net](https://inspirehep.net/files/2e446914ecf6a036ff82d5de6c35a1dd#:~:text=that%20power,%282015%29%20used). Our scalaron is ultralight ($10^{-22}$ eV) and survives to late times as DM. But both are scalar degrees of freedom coupled to curvature. We might extend the action to $f(R,\phi)$ or include a nonminimal coupling $\xi \phi^2 R$ to directly tie $\phi$ to the Ricci scalar. An open question (see §7) is the best coupling form, but qualitatively, the scalaron can act as a **mediator of an extra gravitational interaction** on scales where it is light.
* **Chameleon Screening:** The **chameleon mechanism**​[aanda.org](https://www.aanda.org/articles/aa/pdf/2014/01/aa21061-13.pdf#:~:text=Aims,its%20context%20and%20applied%20it) is a way that a scalar field can evade detection in high-density environments by becoming massive. Our adaptive scalaron is essentially a *cosmological chameleon*. In galaxies (which are relatively dense compared to voids), the scalaron’s amplitude may decrease and its effective mass increase, thereby diluting any “fifth force” it would otherwise produce. The result is that in the solar system or Earth laboratory, the scalaron’s effects (if any) are hidden, consistent with no deviation from Newton/Einstien observed. But in intergalactic space or galaxy outskirts, the field is light and could mediate a force. This is analogous to how chameleon dark energy fields behave – but here the field also carries the dark matter energy density. The work of Salzano *et al.* (2014) directly explored a scalar field with **mass and coupling that scale with local properties** (inspired by chameleon, symmetron, etc.) and found it can fit galaxy dynamics as an alternative to dark matter​[aanda.org](https://www.aanda.org/articles/aa/pdf/2014/01/aa21061-13.pdf#:~:text=Aims,its%20context%20and%20applied%20it)​[aanda.org](https://www.aanda.org/articles/aa/pdf/2014/01/aa21061-13.pdf#:~:text=Results,of%20galaxies%2C%20show%20evident%20correlations). In our scenario we have both: the scalar provides the mass (dark matter) *and* an extra attraction (modified gravity) in varying proportions.
* **MOND-like Behavior:** One might wonder if the scalaron can reproduce the phenomenology of MOdified Newtonian Dynamics (MOND) in galaxies – the empirical scaling where $g\_{\rm obs} \approx \sqrt{g\_{\rm bar} a\_0}$ at low accelerations. Pure fuzzy dark matter generally does not yield MOND-like behavior; it still requires the presence of dark mass to explain rotation curves. However, in some regimes, a scalar-mediated force on baryons could effectively augment gravity. For instance, in Khoury’s superfluid DM, the condensate’s phonons create a $1/r$ force that adds to Newtonian gravity to give MOND’s form​[scipost.org](https://scipost.org/submissions/2109.10928v1/#:~:text=the%20last%20few%20years,studied%20physics%20of%20superfluidity). In the adaptive scalaron context, one could speculate that if a galaxy’s halo is partially in a condensed state, collective excitations might couple to baryons. This is beyond our base model (which has $\phi$ only gravitationally coupled), but if $\phi$ coupled to baryons with a strength $\beta$ (like a scalar–matter coupling), it would introduce a fifth force $F\_\phi = \beta \nabla \phi$. If that force becomes significant in the outer parts of galaxies (where $\phi$ might still be ambient and not too heavy), it could mimic MOND. We won’t derive this here, but simply note that *different effective theories can emerge from the scalaron in different limits*. In one limit (many quanta, decohered), it’s CDM; in another (condensed, excited states), it might approximate a scalar-tensor modified gravity law. The richness of scalar field models allows, in principle, a single framework to produce both phenomena.
* **Relativistic Effects and $f(R,T)$ Gravity:** Some modified gravity theories also include dependencies on the stress-energy trace $T$ (as in $f(R,T)$ gravity) or other invariants. The adaptive scalaron could be extended similarly: its effective potential or mass might depend on $T$ (local matter density). This would explicitly implement the adaptive behavior: $m\_\text{eff}^2(\phi) = m^2 + \alpha T$ for example, where $T$ large (high density) gives a large $m\_\text{eff}$. The result is like a built-in screening. This approach is in line with modern scalar-tensor models that ensure they pass solar system tests by design. In our context, it’s a way to parametrize “the scalaron knows about its environment.”
* **Gravitational Constant Variations:** If the scalaron has a nonminimal coupling, it can cause the *effective gravitational constant* $G\_{\rm eff}$ to vary. For example, in Brans-Dicke theory (scalar-tensor theory), $1/G\_{\rm eff}$ is replaced by a scalar field that can vary in space and time. An ultralight scalaron could cause subtle variations: perhaps between voids and clusters, or from early to late times. Current observations (like primordial nucleosynthesis, CMB, etc.) strongly constrain any such variations, so the scalaron coupling must likely be small. But it’s an avenue where cosmology and local gravity tests cross-constrain the model.

Overall, the adaptive scalaron sits naturally in the scalar-tensor modified gravity landscape. It can be viewed as a *unifying field that carries matter (dark matter) and mediates modified gravity as needed*. It embodies the philosophy that instead of separate dark matter particles and a modification to gravity, a single scalar field’s behavior changes with scale: in cosmic voids or low-acceleration regions it effectively strengthens gravity (since it’s a source of extra attraction distributed more smoothly than clumpy DM), and in high-density regions it behaves just like dark matter (clustered mass that simply contributes to gravity in the usual way). This unity is a significant conceptual simplification, though realizing it quantitatively requires careful model-building to satisfy all constraints.

**Quantum Coherence, Decoherence, and Classical Transition**

The scalaron provides a concrete case study in the quantum-to-classical transition in cosmology. It literally *has its foot in both worlds*: a quantum wave on large scales and a classical particle ensemble on small scales. This duality is typically hard to address in cosmological structure formation, but here it is front and center.

* **Quantum Origin of Structure:** In inflation theory, quantum fluctuations are the seeds of cosmic structure. Likewise, in our model the initial fluctuations of the scalaron field (perhaps generated during inflation or during its misalignment production) are quantum in nature. However, once inflation ends and the universe expands, these perturbations are often treated classically (as in standard perturbation theory). The scalaron’s wave description validates this for large scales while retaining a *literal quantum treatment* for small scales when needed. Essentially, it says: we don’t need to artificially add random phases to mimic classical perturbations – the theory itself will decohere and behave classically at the appropriate point, due to many-body interactions.
* **Decoherence in the Dark Sector:** Usually, when we think of quantum decoherence, we consider particles interacting with photons or a heat bath. Here, remarkably, **gravity itself (and self-interaction of the field) provides the decohering “bath.”** The scalaron in a rich environment (galaxy) acts like a particle system because its wavefunction has effectively collapsed (not via an observer, but via chaotic dynamics). This is a more objective notion of wavefunction collapse – environment-induced decoherence without measurement​[en.wikipedia.org](https://en.wikipedia.org/wiki/Arrow_of_time#:~:text=match%20at%20L436%20decoherence%20is,In%20the%20language). In principle, one could compute a density matrix for the scalaron field, integrate out small-scale modes, and see that the off-diagonal terms (coherences) vanish over time, leaving a diagonal mixture corresponding to various classical field configurations (which is how an N-body system would appear).
* **Re-coherence and Reversibility:** Once decohered, could the scalaron ever re-cohere? In normal circumstances, no – that would require entropy decrease. For example, two decoherent halos merging won’t produce a coherent larger halo; instead, they’ll make an even more chaotic one. Only in very special situations (perhaps at the centers of halos, during a very slow accretion without disturbance) can coherence be partially preserved (hence the persistence of soliton cores). Another interesting thought: If two identical solitonic cores merge head-on with precisely the right phase difference, they might interfere and form a single larger soliton (coherent merger). Simulations indicate solitons can merge and settle into a bigger soliton if conditions are ideal. This is like two Bose condensates merging – if phases align, one gets a condensate; if not, there could be waves and turbulence (decoherence). Thus, **phase alignment matters**. Structure formation is random enough that widespread phase alignment is rare, so typically coherence is local (within each core separately). The arrow of time argument suggests the universe favored the low entropy route initially (one phase everywhere) but as different regions went out of causal contact or collapsed at different times, their phases became uncorrelated.
* **Macroscopic Quantum Effects:** One might ask: are there any observable quantum effects remaining on large scales? Possibly subtle ones. For instance, the fluctuations (granules) in halos are essentially a macroscopic quantum interference pattern. They might cause *random walks* of stars or gas (as the potential fluctuates). This could in principle be distinguished from classical shot-noise fluctuations. Another effect: the very existence of stable solitonic cores is a macroscopic quantum phenomenon (no classical particle collection would remain so condensed without thermal pressure or anisotropy causing collapse or cusp formation). So observing a stable dark matter core in a small galaxy is effectively observing a quantum effect on kiloparsec scales.
* **Connection to Many-Worlds and Measurement:** Philosophically, one could connect this to interpretations of quantum mechanics. In Many-Worlds terms, as the scalaron decoheres, the “branching” of the wavefunction could correspond to different classical halo configurations. But since we only observe one classical outcome, it’s as if the wavefunction’s possibilities have branched into different classical histories – one of which we follow. This is analogous to measurement, but happening in the cosmological context. While deep interpretation is beyond our scope, it’s fascinating that **cosmic structure formation could be viewed as a continual measurement-like process on the dark matter field**, collapsing it into classical structures.

In summary, the scalaron framework doesn’t just bridge quantum and classical physics conceptually; it demands we treat them in one unified description. It stands as a concrete model for how the classical universe we see (galaxies, clusters, black holes) could emerge from an underlying quantum wave reality, with decoherence providing the link – and it does so in a way that is quantitatively testable via astrophysical phenomena like those outlined in §6.

**Twistor Theory and Geometric/Topological Encoding**

Finally, we turn to a more speculative but profound connection: the relation of the scalaron field to **twistor theory and topological structures** in spacetime. Twistor theory, originated by Penrose, seeks to recast physics in a framework where fundamental objects are described in a complex projective space (twistor space) rather than spacetime, potentially uniting quantum theory and gravity​[pos.sissa.it](https://pos.sissa.it/323/003/pdf#:~:text=Broadly%20speaking%2C%20twistor%20theory%20is,as%20some%20of%20its%20historic). One might ask: *what does an adaptive scalaron look like in twistor space?* And can twistor ideas help encode the field’s “memory” – the information carried through its quantum-to-classical evolution?

* **Twistor Representation:** In twistor theory, solutions of field equations in spacetime can be encoded as geometric data (like holomorphic curves) in twistor space​[pos.sissa.it](https://pos.sissa.it/323/003/pdf#:~:text=Broadly%20speaking%2C%20twistor%20theory%20is,as%20some%20of%20its%20historic). For example, certain gravitational or Yang-Mills solutions correspond to algebraic curves in twistor space. A scalar field such as $\phi(x)$, especially if massless or light, could similarly be represented in twistor space (massless scalar fields correspond to certain cohomology classes in twistor space, whereas a massive scalar might require an extension of the twistor formalism). The *non-local relationship* between spacetime and twistor space​[pos.sissa.it](https://pos.sissa.it/323/003/pdf#:~:text=Broadly%20speaking%2C%20twistor%20theory%20is,as%20some%20of%20its%20historic) suggests that some global properties of the scalaron (like phase angles winding around large-scale structures, or topological phase defects if any) could be easier to analyze in twistor space.
* **Phase and Topology:** The scalaron’s phase pattern in space can have topological features. For instance, interference fringes and solitons can be associated with phase discontinuities. If the scalaron field ever had vortices or domain-like structures (not typically in fuzzy DM, but conceivable with certain initial conditions or multiple fields), those would be topologically nontrivial. Twistor theory, being adept at encoding topological and analytic properties of fields, might encode a **global phase configuration as a holomorphic object**. One could imagine that even if the scalaron loses local coherence, some imprint of its initial coherent phase distribution might be stored in a global twistor function (since twistor space can sometimes capture information that is obscured after decoherence in spacetime).
* **Memory of the Wavefunction:** When the scalaron collapses into a black hole, we encounter the question of information loss. Is the information in the scalaron field (the detailed phase-space configuration) lost to the singularity, or is it somehow encoded in subtle correlations (like Hawking radiation correlations or quantum gravity states)? While a full answer requires quantum gravity, twistor theory might provide a language to discuss what is preserved. For example, **gravitational wave memory** is an effect where after gravitational waves pass, detectors remain displaced – the spacetime has a “memory.” By analogy, a scalar field collapse might leave a memory in the surrounding spacetime (perhaps a permanent offset in the scalar field at infinity or some gravitational imprint). Twistor methods, which naturally handle radiation and infinity (via Penrose’s conformal compactification and null infinity structures), could be useful to formalize any such memory. If one were to compute the twistor space description of a collapsing scalaron configuration, one could potentially track what part of the twistor data goes “inside” (lost) versus what remains accessible at infinity.
* **Unification of Descriptions:** One dream is that twistor theory could unify the quantum field aspects and gravitational aspects of the scalaron in one geometric picture​[pos.sissa.it](https://pos.sissa.it/323/003/pdf#:~:text=Broadly%20speaking%2C%20twistor%20theory%20is,as%20some%20of%20its%20historic). In twistor space, a solution that at one limit looks like a linear wave and at another limit like a nonlinear gravitational shock might be represented continuously. For instance, perhaps the transition from a diffuse scalar field to a black hole corresponds to a deformation of a certain twistor space contour. This is highly speculative, but aligns with Penrose’s view of twistor theory as a path to quantum gravity​[royalsocietypublishing.org](https://royalsocietypublishing.org/doi/10.1098/rspa.2017.0530#:~:text=Twistor%20theory%20at%20fifty%3A%20from,general%20relativity%20and%20quantum%20mechanics). As the scalaron is a prime example of a quantum field that significantly affects geometry, it’s a natural playground for such ideas.
* **Twistor and Adaptive Field:** If we were to push the analogy, one might consider the scalaron as an *order parameter* in a twistor-based state. Topologically, the adaptive scalaron may ensure that certain invariants (like winding numbers or indices) remain unchanged even as it transitions. This could address “memory encoding”: maybe the field’s initial conditions are not entirely erased but partly stored in global quantities (like integrals of motion). Twistor space might make those integrals manifest. For example, the integrals of $\phi$ over large 3-volumes, or certain helicity components, might correspond to conserved charges in twistor space.

These ideas are admittedly on the frontier between established theory and conjecture. However, including them underlines the **comprehensiveness of the adaptive scalaron’s reach**: it encourages dialogues between cosmology and more abstract mathematics. Even if twistor methods are not yet fully applied to this scenario, the mere fact we can discuss them indicates the depth of the scalaron concept.

In conclusion of this section, we see that the adaptive scalaron is not an *ad hoc* construct but rather sits at a nexus of multiple frameworks. It behaves like dark matter yet has features of a modified gravity scalar; it originates from quantum physics but explains classical structures; and it even invites consideration in advanced formalisms like twistor theory. This unity is a strong sign that we may be capturing a piece of fundamental truth – that **a single field can weave together the quantum microphysics and the geometric macrophysics of our universe**.

**Observational Signatures and Testable Predictions**

A theory that unifies concepts must ultimately face the test of observation. The adaptive scalaron model makes a variety of **predictions**, many of which are distinguishable from standard $\Lambda$CDM or other new physics. We outline key observational targets, each corresponding to one of the scalaron’s characteristic behaviors:

* **Soliton Core Oscillations and Dynamics:** One striking prediction is the presence of **solitonic cores** in dark matter halos that can **oscillate** and **wander** slightly. In a galaxy with no central black hole, the scalaron core (of order hundreds of parsecs in dwarfs, smaller in bigger halos) should exhibit small-amplitude pulsations (density oscillating by a few percent) and a random walk about the halo center​[osti.gov](https://www.osti.gov/biblio/1851580#:~:text=Previous%20investigations%20found%20that%20the,of%20the%20excited%20states%2C%20the). These might be detectable via their gravitational influence on stars or gas. For example, star motions at the center of a dwarf galaxy might show an additional random velocity dispersion or an oscillatory component not explainable by stellar processes. In the Milky Way, if our inner halo hosts a scalaron core (likely truncated by the central BH’s gravity), it might cause subtle perturbations in the orbits of stars near the galactic center or possibly contribute to the observed Central Molecular Zone kinematics. Observationally, one could search for a **periodic change in the gravitational potential** of a dwarf galaxy core (maybe in systems like Eridanus II which might host a large dark matter core). Precision stellar stream or pulsar timing in the core region could also probe this. If we ever observe multiple snapshots of a dark matter-dominated core (e.g. through strong gravitational lensing over time), we might witness the core’s oscillation. Additionally, the **random walk of the core** could heat the central star cluster of a dwarf galaxy (stars gain energy when the dark potential moves around). This has been suggested as a way to test fuzzy dark matter: look at star clusters in dwarf galaxies for signs of dark matter-induced heating​[osti.gov](https://www.osti.gov/biblio/1851580#:~:text=,deducing%20constraints%20from%20stellar%20heating).
* **Tidal Disruption and Halo Substructure Patterns:** Because scalaron subhalos are less resilient, the pattern of **tidal streams and satellite disruptions** in our galaxy should differ from CDM predictions. In CDM, long thin stellar streams (like GD-1 or the Pal 5 stream) are expected to be punctured and perturbed by numerous subhalo flybys, creating gaps or wiggling the stream. The scalaron model predicts **fewer and gentler perturbations**. Instead of dozens of heavy subhalos carving the stream, we might see only the effect of one or two larger perturbers (the ones that survived) plus a more diffuse, wave-like perturbation from the granules of the dark matter halo. There is tentative evidence that observed streams *do not* show as many small-scale perturbations as a rich CDM subhalo population would produce. This absence is consistent with either a lack of subhalos (as fuzzy DM provides) or subhalos that are too weak to perturb (also the case here). One could quantify this by analyzing stream density power spectra: the adaptive scalaron model predicts a suppression of power on scales corresponding to subhalo masses below $\sim10^8 M\_\odot$, but perhaps an enhanced *continuous* perturbation from interference patterns on larger scales (kpc). Another signature is **tidal dwarf galaxies or orphan cores**: if a subhalo’s outer part is stripped, a lone soliton core might remain as a free-floating object. These would be difficult to detect, but perhaps through gravitational lensing or as microhalo perturbations of cold stellar streams. The existence of an unusually dense, small substructure with no extended halo would be a smoking gun for a soliton remnant. On cluster scales, one might look at **distribution of substructure** via strong lensing: fuzzy/scalaron DM yields fewer small mass clumps, so lensing anomalies (like flux ratio anomalies in lensed quasars) would be less frequent than in CDM. Current lensing constraints are starting to push into this regime, and a confirmed lack of small clumps (or core sizes in clumps) could favor the scalaron idea.
* **Scalar Wave “Echoes” and Gravitational Waves from Collapse:** As discussed, if a scalaron core collapses into a black hole, it can emit a burst of radiation. While direct detection of scalar waves is infeasible (they interact very weakly with detectors unless they mix with photons), gravitational waves could be observable. The model predicts **high-frequency gravitational wave bursts** from such events​[link.aps.org](https://link.aps.org/doi/10.1103/PhysRevD.104.103009#:~:text=%28FRBs%29,more%20evidence%20for%20the%20axion). These could occur for example at the moment a supermassive black hole forms at a galactic center (different from standard stellar collapse; this is dark matter collapse). The frequency could be extremely high (kHz or more), which is outside the band of LIGO but potentially in range of future detectors or resonant bar detectors. One might also search for an **astrophysical signature** of the collapse: e.g., if scalar radiation is emitted, does it couple to any standard model particles? If the scalaron is an axion, it could resonantly convert to photons in magnetic fields, possibly leading to a radio or X-ray flash. Some speculative connections even suggest axion mini-clump collapses might explain fast radio bursts (FRBs) by photon coupling​[link.aps.org](https://link.aps.org/doi/10.1103/PhysRevD.104.103009#:~:text=The%20axion%20objects%20such%20as,secondary%20gravitational%20wave%20production%20associated) – though that requires special conditions. Gravitational wave **“echoes”** are another idea: if after a black hole merger there’s a remnant scalar cloud or exotic structure, one could see repeated echo-like signals in the ringdown. Observers have looked for echoes in LIGO data (none confirmed yet), but the scalaron model provides another context: a newly formed BH (from scalar collapse or merger) could have a quasi-bound scalar wave that leaks out, generating a series of diminishing “echo” pulses in GWs. Detecting such a pattern would hint at physics beyond the vacuum Kerr BH paradigm, possibly supporting the idea of scalar field remnants.
* **Black Hole Scalar Hair and Deviations:** Although a fully formed isolated BH will lose scalar hair, **during formation and in equilibrium with scalar surroundings, there could be observable effects**. For instance, the presence of a scalar field around Sgr A\* (the Milky Way’s BH) could be tested by precision measurements of star orbits (the scalar field’s extra mass or fifth-force could cause anomalous precession). So far, general relativity with a point mass fits S2’s orbit well, which constrains any extended mass (like a fuzzy core) to be quite small. This is consistent with the scalaron expectation that in a massive galaxy, the soliton core radius might be only tens of pc or less, and with Sgr A\*’s $4\times10^6 M\_\odot$ BH present, the core might be largely accreted or disturbed. Still, future telescopes (like a pulsar timing array using a pulsar orbiting Sgr A\*) could detect even small deviations. **Testing no-hair theorem**: Some modified gravity or dark matter models allow black holes with hair. If, say, a stable scalaron configuration (a boson star) coexists or oscillates around a BH, one might detect deviations in the gravitational potential at horizon scales – for instance, the shape of the black hole shadow in EHT observations could differ. At present, M87\*’s shadow was consistent with GR, limiting any extensive scalar structure. But these are still crude tests. Another angle: **binary black hole mergers**. If dark matter around binaries is significant, the merger waveforms could be affected by dynamical friction or scalar radiation. LIGO/Virgo waveforms so far show no clear need for additional effects, implying that by the time of merger, either the scalaron around each BH was negligible or its effects are below detection. Still, a future space-based GW observatory (LISA) observing massive BH mergers might see disparities that hint at a scalar field influence (like a differing energy loss).
* **Halo Density and Structure Patterns:** The core–halo relation in fuzzy scalaron DM is very specific: core mass and halo virial properties are linked (e.g., $M\_{\rm core} \propto (M\_{\rm halo}/10^9M\_\odot)^{1/3}$ roughly, from simulations). Observationally, this means there should be a correlation between the inner dark matter density of galaxies and their outer halo mass. Some data on dwarf galaxies shows a trend of core size with halo mass consistent with FDM expectations, but it’s still debated. If future surveys map many dwarf galaxy rotation curves and get reliable halo masses (perhaps via outer halo tracers or lensing), they could confirm or refute this relation. **A soliton–halo relation** is a unique prediction​[osti.gov](https://www.osti.gov/biblio/1851580#:~:text=We%20report%20that%20a%20fuzzy,As%20the); CDM has no such fixed relation (core properties vary widely depending on baryonic feedback, etc.). Additionally, the **absence of cuspy halos at low mass** and the **flattening of the concentration–mass relation** below a certain scale are predictions we can test with upcoming telescopes (JWST finding dwarf halos at high $z$, or 21-cm surveys mapping small halo gas).
* **Direct Detection and Oscillations:** If the scalaron is an axion, it has a very low mass, meaning its field oscillates at a frequency $m \sim 10^{-22}$ eV $\approx 3\times10^{-8}$ Hz (period ~1 year). This homogeneous oscillation could, in principle, produce a tiny periodic variation in gravitational potentials or constants. Some have proposed searching for axion DM via precision atomic clocks or resonant masses looking for an oscillating signal at the mass frequency. Given our scalaron has such a low frequency, this is challenging (year-long periods, tiny amplitude). But concepts like atom interferometers in space might someday detect a coherently oscillating cosmic field. If detected, that would be a direct evidence of the scalaron (and would measure $m$ outright). While highly futuristic, it’s good to note that the scalaron DM could be probed in labs if its coupling to standard model exists (like the axion-photon coupling, which experiments such as ADMX and others try to exploit, though those are tuned to higher frequencies for heavier axions).

To summarize, the adaptive scalaron framework offers a rich menu of observational tests: from **core dynamics in dwarf galaxies** and **satellite stream perturbations**, to **gravitational wave bursts** and **exotic black hole effects**. Many of these are within reach of current or near-future facilities. A positive detection of any (e.g., discovery of a dark matter soliton through stellar kinematics, or signs of suppressed substructure, etc.) would bolster the case for this theory. Conversely, if observations continue to match CDM on all scales (including small scales), it will tightly constrain the allowed parameter space of an ultralight scalaron (as already, Lyman-$\alpha$ forest data suggests $m \gtrsim 10^{-21}$ eV, or else too much small-scale suppression). Thus, RFT 9.0 not only provides theoretical consistency but also yields **concrete predictions that can falsify or verify the scalaron hypothesis** in the coming years.

**Open Questions and Future Directions**

While the adaptive scalaron model is comprehensive, it also raises many questions and avenues for further research. We highlight several open theoretical questions and goals for future simulations and studies:

* **Scalaron–Curvature Coupling (f(R, φ)) Structure:** What is the most natural and self-consistent way to include the scalaron in the gravitational action? We introduced it minimally, but one could have an action $S = \int d^4x \sqrt{-g},f(R,\phi)$ that directly ties $\phi$ to curvature. A specific form might reproduce the adaptive behavior (for instance, $f(R,\phi) = R + \alpha \phi^2 R - m^2 \phi^2 - \lambda \phi^4 ...$). The open question is: *can we derive the effective environmental dependence of $\phi$’s mass or coupling from a fundamental $f(R,\phi)$ form?* This might require nonlinear analysis of field equations to see how local curvature (or stress-energy) influences the scalaron mass. Work in scalar-tensor theory could guide this, but our scenario is complicated by the need for ultralight mass and cosmic coherence. Another sub-question: how does the scalaron interact with the trace of the stress tensor $T$? In an $f(R,T)$ theory, one could incorporate matter effects explicitly. The goal would be a unified Lagrangian that in the Einstein frame yields a potential for $\phi$ that automatically has a density-dependent minimum (like chameleon). Solving this could make the theory more predictive (fewer free functions to hand-tune the adaptation).
* **Scalaron and Torsion / Emergent Gravity:** Could the scalaron be related to spacetime torsion or an emergent metric phenomenon? In Einstein–Cartan theory (a GR extension including torsion), a scalar field can induce a torsion field or be influenced by it​[sciencedirect.com](https://www.sciencedirect.com/science/article/abs/pii/S000349161830099X#:~:text=Einstein%20gravity%20with%20torsion%20induced,We). Some modern approaches, like teleparallel gravity, allow a **scalar-torsion** coupling (where gravity is encoded in torsion instead of curvature, and a scalar interacts with that)​[link.aps.org](https://link.aps.org/doi/10.1103/PhysRevD.101.024017#:~:text=Post,curvature%20gravity). Exploring a **$f(T, \phi)$** theory (teleparallel equivalent of $f(R, \phi)$)​[sciencedirect.com](https://www.sciencedirect.com/science/article/pii/S0370269324005264#:~:text=gravity%20www,is%20the%20canonical%20scalar%20field) might provide alternative insights – for instance, torsion could screen the scalar in dense regions differently. Also, emergent gravity ideas (like Verlinde’s) suggest that what we call “dark matter” might emerge from microscopic dof. The scalaron might be a more concrete realization: perhaps an underlying topological field in a higher-dimensional or quantum gravity theory that *emerges* as both geometry and matter in 4D. This is speculative, but future theoretical work could try embedding the scalaron in string theory or holography. Is there a dual description where the scalaron’s phases correspond to something like topological braids or twistors? If so, it could connect to quantum error-correcting codes or other modern quantum gravity concepts where “memory” is stored in nonlocal correlations. **Emergent metric models**: Another path is to consider if the metric itself could be an emergent composite of the scalaron (and perhaps other fields). For example, some bi-metric theories or condensate theories (like superfluid vacuum theory) conceive that spacetime might be a condensate of underlying fields. The scalaron as a cosmic BEC could hint that spacetime’s properties (like inertia or the speed of light in vacuum) might slightly change in regions of different scalaron density. No evidence of that exists, but it’s a theoretical curiosity – an environment-dependent scalar field could lead to environment-dependent effective metric for matter (like varying $c$ or $G$ in extreme cases). Exploring consistency of such ideas with precision tests would be needed.
* **Scalaron Collapse and Wavefunction Memory Encoding:** We touched on the fate of information in scalaron collapse. This remains an open problem: *does the scalaron’s wavefunction leave any imprint after forming a black hole?* Hawking’s semiclassical argument suggests the information is lost (or at least not visible in Hawking radiation, leading to the information paradox). In quantum gravity, likely information is preserved in correlations. If the scalaron is fundamental, then presumably unitarity applies to it as well – meaning the full quantum state evolution (scalaron + gravity) is unitary. How is the initial pure state of a soliton encoded in the outgoing Hawking radiation or the final BH state? One speculation: because the scalaron was a coherent state, maybe the black hole’s quantum state reflects that (e.g., a specific phase space distribution of microstates). Is there perhaps a subtle way that the *global phase or some quantum number of the scalaron survives*? For instance, if the scalaron had a global U(1) (like particle number), then black hole no-hair would mean it all went in. But maybe cosmic scalaron fields don’t have a strictly conserved number (axion number is broken by mass term). Still, baryon number sometimes is considered to possibly leave imprint (like a BH might carry a gravitational coupling to global charges). For future work: simulate a scalar field collapse in full GR (numerically) and track entropy of the field vs. area increase of BH – do we see unitary evolution? Also, could **gravitational memory** (permanent metric changes from wave emission) encode some of the information? This ties to our twistor discussion – maybe the outgoing gravitational wave carries a memory that is mathematically related to the initial scalar configuration. Unlocking this could provide a toy model for resolving information paradox in a simplified setting.
* **Phase Maps and Dynamic Transitions:** We currently qualitatively describe “phases” (quantum vs classical) – can we make a **phase diagram** for the scalaron? For example, axes could be local mass density vs. scalaron de Broglie wavelength (or velocity dispersion), and one could delineate regions: “coherent BEC”, “incoherent wave soup”, “classical collisionless”, “unstable collapse”. Having such a diagram would help identify in a given astrophysical context (e.g. at radius X in a halo of mass Y, or at redshift Z) which regime applies. To make it precise, one might define an order parameter for coherence (like the fraction of mass in the ground state mode). Simulations can measure this fraction in halos of various masses and redshifts, to empirically map the transition. Likewise, dynamic processes: how does a region transition from one regime to another? For example, as a dwarf falls into a cluster, its scalaron core might feel increasing tidal field – does it gradually decohere more (exciting more modes), or perhaps if stripped down to just a core, does it *re-cohere* in isolation? Investigating **time-dependent transitions** (through controlled simulation experiments where environment density is changed) would shed light on the hysteresis or irreversibility of these phase changes. Another open point is whether **mixed models** (part scalaron, part ordinary CDM) produce new phenomena – e.g., a fraction of dark matter as scalaron could still form a small soliton in the center and thus give cores, while CDM dominates outer halo. Does this mix solve issues that pure fuzzy DM might have with, say, galaxy counts? Studies of mixed FDM+CDM show intermediate outcomes​file-pbs5tcrmsvz7ndprsed51h; it would be interesting to see if RFT cosmology can accommodate multi-field scenarios as a generalization (RFT 9.1 perhaps, if pure scalaron were falsified​file-pbs5tcrmsvz7ndprsed51h).

These questions define a research program going forward. They involve deepening the theoretical underpinnings (unifying the scalaron with gravity at the action level, possibly involving torsion or emergent ideas) and using simulations and analyses to sharpen the predictions (especially around the quantum-classical boundary and end-state collapse). As RFT cosmology progresses, answering these will tell us if the adaptive scalaron is merely an effective proxy or a fundamental component of the universe’s fabric.

**Conclusion**

In this RFT 9.0 report, we have assembled a comprehensive theoretical picture of the **adaptive scalaron** – a single scalar field hypothesis that spans the gamut of cosmic phenomena from quantum wave-like dark matter to classical halos to modifications of gravity in the strong-field limit. We defined the scalaron’s role as an environmentally-responsive field, motivated by ultralight axion physics and scalar-tensor gravity, which inherently unifies what would otherwise be separate new physics (dark matter particles *and* modified gravity paradigms). Through detailed examination of its three behavioral regimes, we saw that the scalaron can naturally account for structure formation and galactic dynamics: forming quantum-supported solitonic cores in low-density systems, behaving as pressureless dark matter in large halos, and eventually collapsing to black holes under extreme conditions – all while being one and the same entity throughout cosmic history.

This framework offers elegant resolutions to longstanding issues (cusp-core, missing satellites) by attributing them to quantum properties of dark matter, and it does so without sacrificing concordance with high-$z$ structure or precision gravity tests thanks to the chameleon-like adaptation of the field. We also linked the scalaron concept to fundamental physics: the increase of entropy and arrow of time in structure formation aligns with the decoherence of the scalaron’s wavefunction, illustrating on a cosmological stage how classical reality can emerge from quantum beginnings. Moreover, by contemplating connections to twistor theory and emergent gravity, we placed the scalaron idea in a broader context that touches quantum gravity – an enticing prospect that solving dark matter could illuminate quantum-gravitational mysteries like black hole information.

Finally, we underscored that this is a **testable theory**. Upcoming astronomical observations – from the dynamics of dwarf galaxy cores and star streams in Finally, we underscore that this is a **testable theory**. Upcoming observations – from dwarf galaxy core dynamics and Milky Way stellar streams to gravitational-wave searches and black hole imaging – can probe the distinctive features predicted by the adaptive scalaron model. Detection of kiloparsec-scale dark matter cores, a dearth of small halo substructure, or signatures of scalar field collapse (e.g. high-frequency wave bursts) would lend strong support to this framework. Conversely, finding persistently cuspy halos or an abundance of intact subhalos would challenge the scalaron hypothesis. In this way, *RFT 9.0: The Adaptive Scalaron* not only provides a unifying theoretical foundation bridging quantum, classical, and geometric regimes of gravity, but also charts an empirical path forward. As simulations improve and data arrive, the coming years will determine whether a single adaptive scalar field can indeed be the missing key that ties together dark matter and modified gravity – illuminating the deep connection between the quantum waves that pervade our universe and the cosmic structures shaped by gravity.